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(54) Title: METHOD AND APPARATUS FOR IMPROVING RESOLUTION IN SPECTROMETERS PROCESSING OUTPUT STEPS FROM NON-IDEAL SIGNAL SOURCES
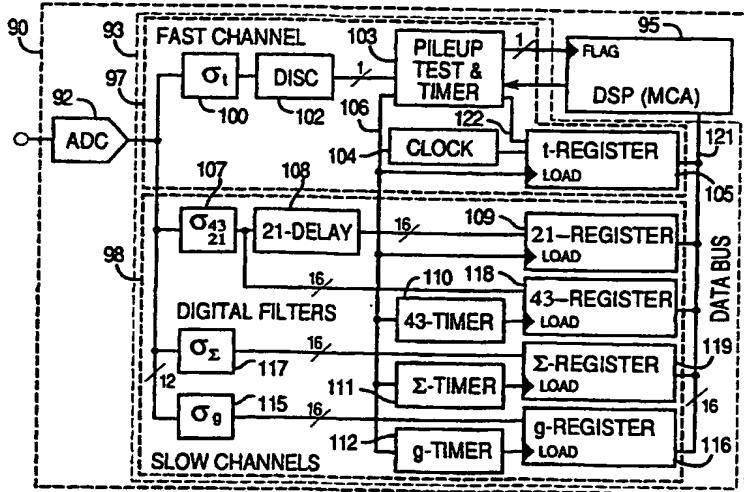
(57) Abstract: A method and apparatus for processing step-like output signals generated by non-ideal, nominally single-pole ("N-1P") devices responding to possibly time-varying, pulse-like input signals of finite duration, wherein the goal is to recover the integrated areas of the input signals is presented. Particular applications include processing step-like signals generated by detector systems in response to absorbed radiation or particles and more particularly, to spectrometers with resistive feedback preamplifiers connected to large volume germanium detectors. Superconducting bolometers can be similarly treated. The method comprises attaching a set of one or more filters (107, 117, 115) to the device's (e.g., preamplifier's) output, capturing a correlated multiple output sample from the filter set in response to a detected event, and forming a weighted sum of the sample values to accurately recover the total area (e.g., charge) of the detected event.

METHOD AND APPARATUS FOR IMPROVING RESOLUTION IN SPECTROMETERS PROCESSING OUTPUT STEPS FROM NON-IDEAL SIGNAL SOURCES

5   CROSS-REFERENCE TO RELATED APPLICATIONS

This application claims priority from provisional application 60/151,020 filed August 27, 1999, the disclosure of which is incorporated by reference in its entirety for all purposes.

10   STATEMENT AS TO RIGHTS TO INVENTIONS MADE UNDER FEDERALLY SPONSORED RESEARCH AND DEVELOPMENT

The U.S. Government has rights in this invention pursuant to Contract No. DE-FG03-97ER82510 awarded by the Department of Energy.

15   BACKGROUND OF THE INVENTION

The present invention relates generally to signal processing systems, and more particularly to processing the step-like output signals generated by non-ideal, nominally single-pole ("N-1P") devices responding to possibly time-varying, pulse-like input signals of finite duration, wherein the goal is to recover the integrated areas of the
20   input signals.

The specific embodiments described relate to processing step-like signals generated by detector systems in response to absorbed radiation or particles and, more particularly, to digitally processing such step-like signals in high resolution, high rate gamma ray (γ-ray) spectrometers with resistive feedback preamplifiers connected to large
25   volume germanium detectors. The application of measuring the step-like output signals from γ-ray detector preamplifiers to measure the γ-rays' energies is just a specific example, and is described because this was the area in which the method was first developed.

The techniques that we have developed solve this problem generally, and
30   therefore should not be construed as being limited to this specific application. Any detection system, for example, that produces output current signals that are integrated by charge sensitive preamplifiers could be treated by these techniques, whether the detected quantities are light pulses, x-rays, nuclear particles, chemical reactions, or otherwise. The techniques, in fact, is not limited to "detector systems" *per se*, but are, in fact, general
35   purpose signal processing techniques which may be broadly applied, once understood. The outputs from superconducting bolometers, for example, produce step-like signals that

are readily treated by the invention. The field of gamma spectroscopy, where 0.1% or less makes the difference between a bad and a good detector, however, provides particularly stringent tests of our techniques.

5   The term "step-like signal" also requires some discussion. The output of an ideal single-pole ("1P") device to an ideal impulse (delta) function input, is an infinitely fast rise time followed by an exponential decay whose time constant τ_d is characteristic of the pole. Viewed on a time scale short compared to τ_d, this output will look like a pure step, while, when viewed on a time scale long compared to τ_d, it will look like a pulse. A real 1P device output, however, will have a finite risetime, τ_r, whose

10   duration will be determined both by the nature of the device and, particularly, by the duration of its real input signal. Provided that τ_r is significantly shorter than τ_d, a real 1P device output signal, viewed on a time scale comparable to τ_d, will then show a risetime region, whose shape may be difficult to describe mathematically, followed, after a period comparable to τ_r, by an exponential decay with time constant τ_d. The output of a N-1P

15   device will be similar, with additional distortions. We will refer to such signals, viewed on this time scale, as "step-like".

Gamma-ray (γ-ray) detection requirements

The detection and measurement of γ-ray energies is a well-established discipline whose primary goal is to accurately determine both the number and energies of

20   γ-rays emitted from some target source. The requirements of good energy resolution and high count rate capability usually conflict, however, since count rates are enhanced by increasing detector volume, which increases output signal distortion and so degrades energy resolution. High count rates also degrade energy resolution directly due to practical problems in preamplifier design.

25   ## Description of the problems

The field of γ-ray detection is highly developed. A fairly comprehensive introduction to the state of the art may be found in the volume "Radiation Detection and Measurement, 2nd Ed." by Glenn F. Knoll [KNOLL-1989]. Below we note only the issues relevant to the present invention. In the first section, we discuss how pole/zero

30   cancellation errors introduce a second pole, spoiling the preamplifier's single pole response. In the second section, we examine how the finite input signal duration, in this case due to charge collection, distorts the preamplifier's output from the ideal.

Pole/zero cancellation errors

Fig. 1A shows a typical solid state γ-ray spectrometer comprising a

35   semiconductor detector diode 7 biased by a voltage supply 8 and connected to a preamplifier 10 comprising an amplifier 13 with a feedback capacitor C 15 and resistor R

17. As drawn, preamplifier **10** is a single pole circuit whose response to an impulse (delta function) input is A exp($-t/\tau_2$), where $\tau_2 = RC$ and A is the area under the impulse. Because τ_2 is typically of order 1 ms, which is too long for the following circuits, a pole/zero (P/Z) network **20** cancels the pole at $1/\tau_2$ and replaces it with a pole at $1/\tau_1$, where τ_1 typically is 50 µs. Gain stage **22** then amplifies and buffers the preamplifier's output signal for shaping amplifier **23** which feeds multichannel analyzer (MCA) **24**.

If the time duration of the current pulse arising from the charge deposited in detector **7** by a γ-ray absorption is very short compared to τ_1, the output of stage **22** will be an exponentially decaying step whose amplitude is the pulse integral and proportional to the deposited charge. γ-ray spectrometers are therefore designed to measure these step amplitudes to measure the charge deposited by the absorbed γ-ray. Other forms of radiation, including neutrons, alpha and beta particles, and x-rays behave similarly and their energies are measured the same way.

Commonly, however, both the input's finite duration and the pole-zero circuit's imperfections distort the preamplifier's response, destroying the proportionality between the output step's amplitude and the deposited charge and so degrading the system's energy resolution. Imperfections in P/Z network **20** arise from difficulties in precisely canceling the τ_2 component, leaving a small residual fraction, of order 1-2%, in the output signal. **Fig. 1B** shows a 5% residual τ_2 component for ease of viewing: an exponential decay signal **25** with time constant τ_2, input to the P/Z network **20**, produces either output signal **27** or **29**, depending upon whether the residual τ_2 term is positive or negative.

These τ_2 residuals are particularly bothersome at high counting rates, where each signal step rides upon a τ_2 background from all preceding steps. As these arrive randomly, the resulting baseline bias also fluctuates randomly in time, which the spectroscopy amplifier's baseline restoration circuit cannot track well. These terms, which may only be a few tenths of 1%, become a significant resolution degradation at 1 MeV where 0.05% energy resolution is desired.

Signal risetime fluctuations and ballistic deficit

Fig. 2 shows a preamplifier **10** front end with a cross sectional view of the detector **7** of **Fig. 1**, for the common coaxial geometry. The dashed lines show electric field line within the detector body **30**, which vary considerably with local geometry. Two factors cause charge collection time variations within the detector and thus risetime variations in the preamplifier's **10** signal output: 1) the difference between carrier velocities; and 2) the existence of different path lengths within the detector. RAUDORPH-1982 describes these issues. These risetime variations produce ballistic deficit by two paths, one direct, one indirect. The direct effect is well understood, per

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GOULDING-1988: the output filter's response varies with the time dependent shape of the charge arrival, being the convolution of the two. A trapezoidal filter greatly reduces this effect in the absence of exponential decay.

5   The indirect effect source of ballistic deficit is due to fluctuations in charge loss through the feedback resistor with differing risetime, as seen in **Fig. 3A** with two risetimes, **40** and **42**, where **Fig. 3B** enlarges their peak regions. The slower risetime signal loses less charge and thus is larger once charge collection is complete. Even filters which ignore the charge collection region are still sensitive to this lost charge effect, and relatively small errors of this size can substantially degrade resolution. For a trapezoidal
10   filter, the collection time difference shown **Fig. 3B** produces a 0.2% amplitude difference (2,000 eV at 1 MeV) which will degrade ideal 1.7 keV resolution to 2.6 keV. Ballistic deficit errors must therefore be reduced to less than 0.05% to obtain ideal spectrometer resolutions at 1 MeV.

Charge Trapping Losses

15   Charge trapping also produces errors in γ-ray energy measurements since trapped charges are lost to the measurement. The present invention does not seek to address this problem.

Generalizations

It is important to note that the pole-zero cancellation errors described
20   above do not arise from the preamplifier's use in γ-ray spectroscopy, but are a generic problem in low noise, charge sensitive preamplifiers. Similarly, while the described risetime fluctuations described above were attributed to the geometry and construction of large volume Ge detectors, it is clear that such problems fundamentally arise from interactions between the finite charge collection time and the electrical characteristics of
25   the preamplifier and not from the physics of the collection processes. Geometric heat flow variations in the photon absorbing mass produce similar effects in the superconducting bolometers mentioned earlier. Risetime issues are therefore a potentially general problem as well, and may need to be corrected for in other, non-γ-ray detectors whenever the highest measurement accuracies are required. The methods we describe
30   offer just that capability. Further, the terminology "single-pole" or "multiple-pole" comes from the LaPlace Transform treatment of differential equations describing time variant phenomena. Any device which shows "single pole" behavior, for example, will display exponential time decay in response to an impulse input and, therefore, may be, for example, mechanical, thermal, chemical, or magnetic in nature in addition to the
35   electronic case presented here. Our method can be directly applied to these devices as well, as will be apparent from the teachings herein.

Existing Correction Schemes

The prior art deals with pole/zero errors in two ways: first, by canceling τ_2 as accurately as possible; and, second, with baseline restoration schemes which try to track the shaping amplifier's "no signal" output as closely as possible, an approach which

5   degrades as rates becomes high. We have not found any approaches which measure and/or correct for the effect directly.

Over the years, various heuristic schemes have been developed which attempt to compensate for ballistic deficit. RADEKA-1982 introduced trapezoidal filtering and developed a time-variant implementation, using a gated filter following a

10   semi-Gaussian shaper, that provided significant resolution improvements. WHITE-1988 proposed a different gated integration approach using a series switch to excise the charge collection region out of the preamplifier signal entirely. The final circuit was complex and had enhanced deadtime problems. GOULDING-1988, RAUDORF-1982, and SIMPSON-1990 disclose schemes that depend on directly measuring the signal's risetime

15   t_r and correcting the energy filter output by a term like $t_r{}^n$. These approaches are complex to implement and require precise expert adjustments to operate. The underlying assumptions are not particularly valid and improvements in energy resolution have been modest in practice.

HINSHAW-1991 and KUMAZAWA-1998 describe attempts to correct for

20   ballistic deficit by capturing peak amplitudes from two filters which respond to the ballistic deficit differently, one an energy measuring filter and one a differentiating (or bipolar shaping) filter. Typically a significant fraction of their difference in peak heights is added to the energy filter's peak to correct it.

Related Art

25   There is some related art wherein the details of the shapes of the preamplifier output signals are sampled digitally and used either to distinguish between different types of particles absorbed in the detector (e.g., MILLER-1994) or to distinguish between single and multiple interaction events in large germanium detectors. See, for example, TAKAHASHI-1994 and AALSETH-1998.

30   WARBURTON-1997, WARBURTON-1998, and WARBURTON-1999 describe methods for implementing digital filtering and x-ray spectroscopy. While they do not address the issues under consideration, some of their filtering techniques will be employed in the present invention and are referenced in the specification.

SUMMARY OF THE INVENTION

The present invention provides techniques for measuring a step-like output signal from a nominally single-pole (N-1P) device in response to a pulse-like input signal to determine the integrated area of said input signal. The invention addresses the

5   possibility that the device deviates from ideality due to the presence of additional poles, zeros and/or a DC offset and that the input pulses may have finite time durations and variable amplitudes. In a specific example, the measurement determines collected detector charges from step-like preamplifier output signals in the presence of both risetime fluctuations and imprecise pole/zero cancellations.

10   In brief, the present invention contemplates processing the N-1P device's (e.g., preamplifier's) output signal using a set of one or more shaping filters. Where plural filters are used, they typically have different time constants. A set of samples of the outputs of this filter set is captured in such a manner that the multiple sample values bear prescribed time relationships to one other. We refer to this set of sample values as a

15   correlated multiple output sample (or "cMOS" for short). The relationships among the individual sample values may be determined by the times of their capture, by delay elements inserted in the signal paths, or some combination of the two. Further, the different individual sample values can be obtained from associated different filters, or plural sample values can be obtained from the same filter, but captured at different times

20   or with different delays. The term cMOS is intended to cover these multiple possibilities and is discussed further in § 5.1 below.

The input pulse's integrated area is determined by capturing a cMOS in response to detecting a step-like signal (sometimes called an event). A weighted sum of the individual sample values in the cMOS (sometimes referred to as cMOS values) is then

25   formed as a measure of the input pulse's integrated area (e.g., total charge). The weighting factors can be computed directly from information about the N-1P device's decay constants, the filter set, and the prescribed time intervals in the cMOS.

While embodiments of the invention use triangular and trapezoidal filters, the invention does not require specific filter shapes. Further, the invention does not

30   require that the filter's capture times be precisely located relative to the step-like signal's leading edge. Rather, the method derives its accuracy from repeatably reproducing the set of prescribed time relationships between the sample values in the cMOS. The underlying capture method is therefore time-based rather than amplitude-based, which differentiates the invention from prior art methods which capture filter samples based on

35   their maximum amplitudes.

While the preferred implementation uses digital signal processing, implementations using solely analog processing or hybrid approaches are also feasible. Embodiments of the invention require neither a measurement of the step-like signal's

risetime nor more precise information about its arrival time than is found by the pileup inspection circuits of commonly available commercial shaping amplifiers.

When using a single cMOS, however, the measurement's noise is increased by the number N of additional filter samples required to compensate for the

5   N-1P device's non-ideal terms. For those cases where this increased noise is significant, we also show how N may be reduced by creating a parameterized model of the non-ideal terms; making baseline measurements to determine the parameters; and then using them to correct area (e.g., charge) measurements made using simpler sets of filters which have less noise. In some implementations, these baseline measurements are cMOS values

10   captured at times when the filters are not processing the step-like signals. In the detector-preamplifier case described in detail, the resultant implementation achieves the low electronic noise levels of conventional trapezoidal filters while also eliminating resolution loss due to both ballistic deficit and high count rates.

A further understanding of the nature and advantages of the present

15   invention may be realized by reference to the remaining portions of the specification and the drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

Fig. 1A shows a schematic of a semiconductor γ-ray detector front end, including a resistive feedback integrator, pole/zero network and output gain stage;

20   Fig. 1B shows voltages generated by the preamplifier portion of the circuit in Fig. 1A when the pole/zero circuit is not ideally adjusted;

Fig. 2 expands the detector structure of Fig. 1A to demonstrate the geometric and other factors which contribute to signal risetime variations;

Fig. 3A shows the variation in risetime between pure electron and pure

25   hole collection for a simple collection path in the detector structure of Fig. 2;

Fig. 3B shows an expanded view of the signal peaks in Fig. 3A;

Fig. 4 shows a step-like feature in a general preamplifier output, broken into its component signals, and establishes a nomenclature system for describing these signals mathematically;

30   Fig. 5A and 5B show how to compute the appropriate intervals between capture times of the multiple filters;

Fig. 6A and 6B show a pair of purely digital circuits which implement the described method;

Fig. 7 shows a purely analog circuit which implements the described

35   method; and

Fig. 8 shows a hybrid analog/digital circuit which implements the described method.

DESCRIPTION OF SPECIFIC EMBODIMENTS

1. Introduction

5   The invention has three basic steps: applying a set of filters to the preamplifier output; capturing a correlated multiple output sample ("cMOS") from the filter set in response to a detected event; and forming a weighted sum of these sample values to accurately recover the energy of the detected event. The cMOS is a set of samples bearing a prescribed set of time relationships between one another. The

10   appropriate time relationships are achieved by setting the times of capture, inserting delays into the signal paths, or a combination thereof, and are independent of the specific risetime shape of any particular event. Our overall goal, therefore, is to describe how to select an appropriate set of filters; explain what it means to capture a cMOS and how to achieve it; and show how to compute the weighting functions.

15   The theory underlying the invention method is based on modeling the preamplifier's (or N-1P device's) step-like output response (the "output") to the input charge (or impulse: the "input") generated by the event as the sum of a small number of analytical functions which are represented by the same number of amplitudes. In a specific embodiment, five amplitudes are used to describe: the τ_1 and τ_2 exponential

20   components prior to the step-like signal, the ideal amplitude of the step itself, an error term due to the ballistic deficit, and a DC level. We first reduce the number of amplitudes to four by making differential filter measurements which are DC level insensitive. To find these four amplitudes, we make four independent measurements (from three filters) in the step's vicinity, express each measurement as a linear

25   combination of the four amplitudes, and solve the resultant set of four linear equations to recover the ideal step amplitude, which is proportional to the input charge (or impulse area). Obtaining a set of equations which is well conditioned and insensitive to preamplifier noise, ballistic deficit, and τ_2 amplitude is then primarily a matter of selecting an appropriate set of filters and time correlated capture points, which then define

30   the cMOS.

Step amplitudes obtained using the four-filter measurement method have almost a factor of 3 higher electronic noise than simple trapezoidal filters. In cases where this is important, we show how simpler filters can be corrected for the τ_2 and DC components by developing a parameterized model of these terms and making one or more

35   baseline measurements to determine their values. When multiple measurements are made, the parameters must be compensated for their evolution in time, creating a time-compensated model.

The description below is organized as follows In § 2, using a static, captured step-like signal, we locate several filters in the vicinity of its leading edge and mathematically describe their outputs in terms of the base amplitudes. In § 3, we use these results to set up and solve the set of linear equations to obtain the input charge.

5   This solution has the form of a weighted sum of the filter values, showing the genesis of the weighting coefficients. In § 4, we evaluate the method for two filter sets which compensate simultaneously for inexact pole/zero cancellation and ballistic deficit. In § 5, we introduce the concept of the cMOS to show how to implement the invention in real time processing situations. In § 6, we will describe various circuits which implement this

10   method: a purely digital pair, a purely analog circuit and a hybrid analog-digital circuit. In § 7, we analyze our method's electronic noise. Finally, in § 8, we show how a simpler, three filter measurement can be used to achieve similar accuracy with lower noise, provided time recorded baseline measurements are made to correct for the τ_2 and DC preamplifier components

15   2. Mathematical description of generalized filter outputs

          2.1. Decay components of the step-like signal

              Fig. 4 shows a typical preamplifier step-like output signal as it might look if it were digitized by an analog to digital converter (ADC) and saved in a computer memory. The shown time scale is arbitrary, with zero set a little before the onset of

20   charge collection for convenience. To simplify the image, only a few members of the set of discrete values have been identified explicitly: subscripted capital Q's denote values at specific instants since the signal really represents the charge integrated on feedback capacitor **15** by amplifier **13**. The τ_1 component has three regions: an exponential decay region **50** prior to the step (i.e., times less than 0), a charge collection region **52**, and an

25   exponential decay region **53** following the step (i.e., times greater than 1). The curve **55**, showing how curve **50** would have continued to decay had the step not arrived, represents the continued time decay of all previous steps, that is, the preamplifier's first pole residual response to those previous steps. The signal's τ_2 component has the equivalent four regions **57, 58, 60,** and **62**. Finally, there is the preamplifier's DC offset value B_0 **65**,

30   which is constant. The τ_1 and τ_2 charges injected by the γ-ray pulse decay independently, since the preamplifier is a linear device. The total collected charge Q_{gT} generating this event thus divides into $Q_{1gT} = (1-\beta)Q_{gT}$ for τ_1 and $Q_{2gT} = \beta Q_{gT}$ for τ_2, β being the division ratio.

              It is important to note that, in the following, we will be describing a signal

35   processing technique which relies on the mathematical form of the signals and not on their physical source. While we develop the method in the context of preamplifier signals from γ-ray detector preamplifiers, the method itself can be usefully applied to any signals

which have the appropriate form. Similar outputs would generally be obtained from nominally single pole devices (N-1P devices) in response to impulse inputs of extended duration and variable amplitude. In the present case these signals result from a single pole electronic circuit responding to an input pulse from a detector, in other cases the detector itself may produce signals of the appropriate form. The temperature response of a superconducting bolometer, for example, shows this behavior [STAHLE-1999] and could be treated by the method.

2.2. Running-sum filter output

If r values are sampled at uniform intervals Δt from an exponential decay with time constant τ_j, then the ith value $Q_{j,i}$ is given by:

$$Q_{j,i} = Q_{j,0} \exp(-i \, \Delta t/\tau_j) = Q_{j,0} \, b_j^i, \text{ for } (0 \le i \le r-1) \tag{1}$$

where $b_j = \exp(-\Delta t/\tau_j)$ is the decay per Δt. Therefore, a kth digital filter, defined by weighting constants $\{w_{i,k}, 1 \le i \le r-1\}$, acting on the values in **Eqn. 1** gives:

$$\sigma_{jk,r} = \sum_{i=0}^{r-1} Q_i \, w_{i,k} = Q_{j0} \sum_{i=0}^{r-1} b^i \, w_{i,k} \equiv Q_{j0} A^*_{jk,r} \tag{2}$$

where the * marks this as a general filter result. This shows that the filter output is linear in the amplitude $Q_{j,0}$, multiplied $A^*_{j,k,r}$, which is a constant for a particular filter and decay constant τ_1. For a simple running-sum filter (see WARBURTON-1999), w equals unity, so

$$A_{jr} = \sum_{i=0}^{r-1} b^i_j = \frac{1-b^r_j}{1-b_j} \equiv r_j \left(1 - b^r_j\right) \tag{3}$$

Eqn. 3 also defines the division factor r_j, which is equal to the sum to infinity.

2.3. General trapezoidal filter output with risetime in its gap

Next, consider a running-sum $\sigma_{\Sigma+}$ of m samples starting at time 1 over the signal in **Fig. 4**. Decomposing the signal into 5 components with amplitudes $\{Q_{g13}, Q_{13}, Q_{g23}, Q_{23},$ and $B_0\}$, **Eqns. 2** and **3** give:

$$\sigma_{\Sigma+} = Q_{g13}A_{1,m} + Q_{13}A_{1,m} + Q_{g23}A_{2,m} + Q_{23}A_{2,m} + mB_0 \tag{4}$$

where the index j in $A_{j,m}$ equals 1 or 2 for τ_1 and τ_2. The equivalent sum $\sigma_{\Sigma-}$, starting at time -4, which only sees amplitudes $\{Q_{13}, Q_{23},$ and $B_0\}$ is then:

$$\sigma_{\Sigma-} = Q_{10}A_{1,m} + Q_{20}A_{2,m} + mB_0 \tag{5}$$

The difference $(\sigma_{\Sigma+} - \sigma_{\Sigma-})$ is a trapezoidal filter σ_Σ of risetime m and gap g. Since $Q_{10} = b_1^{-m-g} Q_{13}$ and $Q_{20} = b_2^{-m-g} Q_{23}$:

$$\sigma_\Sigma = \sigma_{\Sigma+} - \sigma_{\Sigma-} = Q_{g13}A_{1,m} + Q_{13}\alpha(1,m,-m-g) + Q_{g23}A_{2,m} + Q_{23}\alpha(2,m,-m-g) \tag{6}$$

where $\alpha(j,r,s) = A_{j,r}(1-b_j{}^s)$ and the DC components cancel. In **Eqn. 6**, we stress that Q_{13} represents, at time instant 3, the preamplifier's dominant pole (τ_1) response to all preceding step-like signal events; Q_{23} similarly represents the secondary pole (τ_2); while Q_{g13} and Q_{q23} are their amplitude responses to the present event. **Eqn. 6** takes its simple form because the signal rise occurs entirely within the filter gap region g, so only decaying exponential terms are measured outside. Thus σ_Σ is relatively insensitive to the precise location of the step within the gap region because its weighting constants are negligible (zero, in fact) there.

2.4. Triangular filter over the charge collection region

To explicitly probe the step's risetime region, we will use a triangular filter (gap = 0) $\sigma_g = (\sigma_{g+} - \sigma_{g-})$ of risetime g, whose subtractive leg σ_{g-} sits precisely within the gap of σ_Σ. (This relationship will also be easy to achieve when we implement a digital real time system.) We will show that the precise location of the step within the gap is not important. Then, as above:

$$\sigma_g = \sigma_{g+} - \sigma_{g-}$$

$$= Q_{g13}A_{1,g} + Q_{13}\alpha(1,g,-g) + Q_{g23}A_{2,g} + Q_{23}\alpha(2,g,-g) - \sum_{i=0}^{g-1} q_{1g,i} - \sum_{i=0}^{g-1} q_{2g,i} \tag{7}$$

where the $q_{jg,i}$ are the sampled charge values at each interval in the subtracted running-sum region, each comprising the new charge increment collected in that interval plus the exponentially decaying remnants of the charges collected in the previous intervals.

Since each charge increment decays to zero independently, while their sum equals the total collected charge, we can write

$$r_1 Q_{1gT} = \sum_{i=0}^{\infty} q_{1g,i} \quad \text{and} \tag{8a}$$

$$r_2 Q_{2gT} = \sum_{i=0}^{\infty} q_{2g,i} \tag{8b}$$

Splitting the infinite summation, however, also gives:

$$r_1 Q_{1gT} = r_1 Q_{gT}(1 - \beta) = \sum_{i=0}^{g-1} q_{1g,i} + r_1 Q_{g13} \tag{9}$$

so that

$$r_1 Q_{1gT} = r_1 Q_{gT}(1 - \beta) = \sum_{i=0}^{g-1} q_{1g,i} + r_1 Q_{g13} \tag{10}$$

The correction charge Q_c defined in **Eqn. 10** is the difference between the total τ_1 collected charge and the amount remaining at the end of gap g. Q_c will typically be of order 2% of Q_{gT}. As a small correction term, Q_c will be relatively uncorrelated to Q_{g13},

so the set of linear equations we solve for Q_{g13} and Q_c be well conditioned under inversion.

Similarly, for the τ_2 component:

$$\sum_{i=0}^{g-1} q_{2g,i} = r_2 \left\{ Q_{gT}\,\beta - Q_{g23} \right\} \equiv r_2\,Q_{c2}, \tag{11}$$

5   where Q_{c2} will be smaller than Q_c by approximately the ratio τ_1/τ_2. **Eqns. 10 and 11** allow us to express σ_g entirely in terms of Q_{gT}, the total charge, and values from outside of the charge collection region, where the signals behave strictly as decaying exponentials. Equivalent integral expressions may similarly be derived in the continuous signal case or for filters with other weighting constants.

10   2.5. Eliminating Q_{c2}

When the gap region is much smaller than both τ_1 and τ_2, Q_{c2} becomes proportional to Q_c to a high degree of accuracy. This may be seen by defining Q_{gi} as the total γ-ray charge collected at time interval i in the collection region. Since each Q_{gi} decays independently, we can write for the two components:

$$\sum_{i=0}^{g-1} (1-\beta)Q_{gi}\, b_1^{g-i} = Q_{g13} \quad \text{and} \tag{12a}$$

$$\sum_{i=0}^{g-1} \beta Q_{gi}\, b_2^{g-i} = Q_{g23}, \tag{12b}$$

while, by definition,

$$\sum_{i=0}^{g-1} (1-\beta)Q_{gi}\, b_1^{g-i} = Q_{g13} \quad \text{and} \tag{13a}$$

$$\sum_{i=0}^{g-1} \beta Q_{gi}\, b_2^{g-i} = Q_{g23}. \tag{13b}$$

20   Thus, for Q_c:

$$Q_c = (1-\beta)Q_{gT} - Q_{g13} = (1-\beta)\sum_{i=0}^{g-1} Q_{gi}(1 - b_1^{g-i}). \tag{14}$$

Expanding b_1^{g-i} to good accuracy as $(1-(g-i)\Delta t/\tau_1)$ gives $(1-b_1^{g-i})$ equal to $(g-i)\Delta t/\tau_1$ so:

$$Q_c = (1-\beta)\frac{\Delta t}{\tau_1}\sum_{i=0}^{g-1} Q_{gi}(g-i). \tag{15}$$

Similarly:

$$Q_{c2} = \beta\frac{\Delta t}{\tau_2}\sum_{i=0}^{g-1} Q_{gi}(g-i), \tag{16}$$

25   where ρ_2 is equal to $\Delta t/\tau_2$. Comparing **Eqns. 15 and 16**:

$$Q_{c2} = \frac{\beta \tau_1}{(1 - \beta)\tau_2} Q_c \equiv FQ_c \tag{17}$$

Q_{c2} is therefore quite small, being about 0.1% of Q_c or 0.002% of Q_{gT}. We can therefore express any of our filters in terms of the four quantities Q_{gT}, Q_c, Q_{g13} and Q_{g23} using:

$$\sum_{i=0}^{g-1} q_{1g,i} = r_1 Q_c \tag{18a}$$

$$Q_{g13} = (1 - \beta)Q_{gT} - Q_c \tag{18b}$$

$$\sum_{i=0}^{g-1} q_{2g,i} = r_2 FQ_c \tag{18c}$$

$$Q_{g23} = \beta Q_{gT} - FQ_c \tag{18d}$$

3. Solving for collected charge Q_{gT} from the captured waveform

3.1. Filter expressions in terms of the base amplitudes

§ 2.5 shows how all difference filters in the vicinity of the step's leading edge can be expressed as linear equations in the four amplitudes Q_{gT}, Q_c, Q_{g13} and Q_{g23} (non-difference filters will also require B_0). Therefore we must make four (or five) independent filter measurements to solve for Q_{gT}. While a wide variety of filters could be used, certain sets will be better conditioned than others and will also require less measurement precision to accommodate step risetime variations. Research and experimentation have shown the following set to function well: 1) the trapezoidal "energy" filter of risetime m from **Eqn. 6**; 2) the triangular "risetime probe" filter of **Eqn. 7**; and, 3) a pair of triangular "τ_1 and τ_2 background" filters of risetime m/2, situated at times -4 and +1 in **Fig. 4**, respectively. It is important to notice that, in this selection, only the risetime probe filter processes the step's risetime, which lies in the energy filter's gap and is entirely outside the range of the two other triangular filters. This placement, where only the risetime probe filter has non-negligible weighting constants in the risetime region, assures that their outputs, and hence Q_{gT} estimate, will be insensitive to the precise location of the signal step within the gap, which, in real time implementations, will remove any need for great timing precision. That the filters share some common starting and stopping points is primarily a matter of mathematical and implementational convenience, but is not fundamental to the technique.

Substituting **Eqns. 18** into **Eqns. 4** through **7** gives four linear equations for $\sigma_{\Sigma+}$, $\sigma_{\Sigma-}$, σ_{Σ}, and σ_g in terms of Q_{gT}, Q_c, Q_{13} and Q_{23}:

$$\sigma_{\Sigma+} = Q_{gT}\Big[(1-\beta)A_{1,m}+\beta A_{2,m}\Big] - Q_c\Big[A_{1,m}+FA_{2,m}\Big] + Q_{13}\Big[A_{1,m}\Big]$$

$$+ Q_{23}\left[A_{2,m}\right] + mB_0 \quad , \tag{19a}$$

$$\sigma_{\Sigma-} = Q_{gT}[0] - Q_c[0] + Q_{13}\left[b_1^{-m-g}A_{1,m}\right] + Q_{23}\left[b_2^{-m-g}A_{2,m}\right] + mB_0 \quad , \tag{19b}$$

$$\sigma_{\Sigma} = Q_{gT}\left[(1-\beta)A_{1,m}+\beta A_{2,m}\right] - Q_c\left[A_{1,m}+FA_{2,m}\right] + Q_{13}[\alpha(1,m,-m-g)]$$
$$+ Q_{23}[\alpha(2,m,-m-g)] \quad , \tag{19c}$$

$$\frac{\sigma_g}{r_1} = Q_{gT}\left[\frac{(1-\beta)A_{1,g}+\beta A_{2,g}}{r_1}\right] - Q_c\left[(2-b_1^g)+\frac{r_2}{r_1}F(2-b_2^g)\right] + Q_{13}\left[\frac{\alpha(1,g,-g)}{r_1}\right]$$
$$+ Q_{23}\left[\frac{\alpha(2,g,-g)}{r_1}\right] \quad ; \tag{19d}$$

We divided σ_g by r_1 so that the coefficient of Q_c will approximately equal unity for inversion stability.

Using the same methods for the outputs of three filters of length $m/2$ (two triangular filters σ_{43} and σ_{21} whose running-sum first points are: $\sigma_{43} = $ [time +3: time +1] and $\sigma_{21} = $ [time -2: time -4] and a trapezoidal filter $\sigma_{32} = $ [time +2: time -2]) gives:

$$\sigma_{43} = - Q_{gT}\left[(1-\beta)\alpha\left(1,\frac{m}{2},\frac{m}{2}\right)+\beta\alpha\left(2,\frac{m}{2},\frac{m}{2}\right)\right] + Q_c\left[\alpha\left(1,\frac{m}{2},\frac{m}{2}\right)+F\alpha\left(2,\frac{m}{2},\frac{m}{2}\right)\right]$$
$$- Q_{13}\left[\alpha\left(1,\frac{m}{2},\frac{m}{2}\right)\right] - Q_{23}\left[\alpha\left(2,\frac{m}{2},\frac{m}{2}\right)\right] \quad , \tag{20a}$$

$$\sigma_{21} = - Q_{13}\left[b_1^{-m-g}\alpha\left(1,\frac{m}{2},\frac{m}{2}\right)\right] - Q_{23}\left[b_2^{-m-g}\alpha\left(2,\frac{m}{2},\frac{m}{2}\right)\right] \quad , \tag{20b}$$

$$\sigma_{32} = Q_{gT}\left[(1-\beta)A_{1,m/2}+\beta A_{2,m/2}\right] - Q_c\left[A_{1,m/2}+FA_{2,m/2}\right]$$
$$+ Q_{13}\left[\alpha\left(1,\frac{m}{2},-\frac{m}{2}-g\right)\right] + Q_{23}\left[\alpha\left(2,\frac{m}{2},-\frac{m}{2}-g\right)\right] \quad . \tag{20c}$$

These filters also exclude the step's rising edge for location insensitivity. While this is not a requirement of our invention, it significantly simplifies both the coefficient derivations and trigger system timing requirements.

3.2. The four-filter solution for Q_{gT}

Collecting results for the four filters listed at the start of § 3.1, we have:

$$\sigma_{\Sigma} = Q_{gT}\left[(1-\beta)A_{1,m}+\beta A_{2,m}\right] - Q_c\left[A_{1,m}+FA_{2,m}\right]$$

$$+ Q_{13}[\alpha(1,m,-m-g)] + Q_{23}[\alpha(2,m,-m-g)] \tag{21a}$$

$$\frac{\sigma_g}{r_1} = Q_{gT}\left[\frac{(1-\beta)A_{1,g}+\beta A_{2,g}}{r_1}\right] - Q_c\left[(2-b_1^g)+\frac{Fr_2}{r_1}(2-b_2^g)\right]$$

$$+ Q_{13}\left[\frac{\alpha(1,g,-g)}{r_1}\right] + Q_{23}\left[\frac{\alpha(2,g,-g)}{r_1}\right], \tag{21b}$$

$$\sigma_{43} = - Q_{gT}\left[(1-\beta)\alpha\left(1,\frac{m}{2},\frac{m}{2}\right)+\beta\alpha\left(2,\frac{m}{2},\frac{m}{2}\right)\right] + Q_c\left[\alpha\left(1,\frac{m}{2},\frac{m}{2}\right)+F\alpha\left(2,\frac{m}{2},\frac{m}{2}\right)\right]$$

$$- Q_{13}\left[\alpha\left(1,\frac{m}{2},\frac{m}{2}\right)\right] - Q_{23}\left[\alpha\left(2,\frac{m}{2},\frac{m}{2}\right)\right], \tag{21c}$$

$$\sigma_{21} = - Q_{13}\left[b_1^{-m-g}\alpha\left(1,\frac{m}{2},\frac{m}{2}\right)\right] - Q_{23}\left[b_2^{-m-g}\alpha\left(2,\frac{m}{2},\frac{m}{2}\right)\right], \tag{21d}$$

as four linear equations in four unknowns. The unknown parameters are of three types: 1) (P_1) the unknown charge Q_{gT} (the area of the input impulse in the general case); 2) (P_2) charges describing the step's finite arrival time (Q_g) and the residual charges in the τ_1 and τ_2 decay modes (Q_{13} and Q_{23}) from previous events; and 3) (P_3) a set of fixed parameters $\{\tau_1, \tau_2, \beta, m \text{ and } g\}$ which describe the preamplifier's (or N-1P device's) transfer function and the applied filters. Since the coefficients in [ ]'s are functions only of the P_3 parameters, they are constants which need be computed only once for a given preamplifier and filter set.

Eqns. 21 can be solved by several methods, including casting them into matrix notation and then inverting the coefficient matrix. Thus:

$$\begin{bmatrix} \sigma_\Sigma \\ \dfrac{\sigma_g}{r_1} \\ \sigma_{43} \\ \sigma_{21} \end{bmatrix} =$$

$$\begin{bmatrix} (1-\beta)A_{1,m}+\beta A_{2,m} & -A_{1,m}-FA_{2,m} & \alpha(1,m,-m-g) & \alpha(2,m,-m-g) \\ \dfrac{(1-\beta)A_{1,g}+\beta A_{2,g}}{r_1} & -(2-b_1^g)-\dfrac{Fr_2}{r_1}(2-b_2^g) & \dfrac{\alpha(1,g,-g)}{r_1} & \dfrac{\alpha(2,g,-g)}{r_1} \\ -(1-\beta)\alpha\left(1,\tfrac{m}{2},\tfrac{m}{2}\right)-\beta\alpha\left(2,\tfrac{m}{2},\tfrac{m}{2}\right) & \alpha\left(1,\tfrac{m}{2},\tfrac{m}{2}\right)+F\alpha\left(2,\tfrac{m}{2},\tfrac{m}{2}\right) & -\alpha\left(1,\tfrac{m}{2},\tfrac{m}{2}\right) & -\alpha\left(2,\tfrac{m}{2},\tfrac{m}{2}\right) \\ 0 & 0 & -b_1^{-m-g}\alpha\left(1,\tfrac{m}{2},\tfrac{m}{2}\right) & -b_2^{-m-g}\alpha\left(2,\tfrac{m}{2},\tfrac{m}{2}\right) \end{bmatrix}\begin{bmatrix} Q_{gT} \\ Q_c \\ Q_{13} \\ Q_{23} \end{bmatrix} \tag{22}$$

becomes
$$\sigma = \mathbf{J} \bullet \mathbf{Q}, \tag{23a}$$

so that
$$\mathbf{Q} = \mathbf{J}^{-1} \bullet \sigma, \tag{23b}$$

and
$$Q_{gT} = J_{11}^{-1}\sigma_\Sigma + J_{12}^{-1}\frac{\sigma_g}{r_1} + J_{13}^{-1}\sigma_{43} + J_{14}^{-1}\sigma_{21}, \tag{23c}$$

As a solution for Q_{gT}, **Eqn. 23c** has the advertized form: a weighted sum of the four filter outputs. The weighting coefficients J_{ij}^{-1} are computed only once for a particular set of filters and P_3 parameters $\{\tau_1, \tau_2, \beta, m \text{ and } g\}$ and then multiplied by the four filter outputs σ_Σ, σ_g/r_1, σ_{43} and σ_{21} to obtain the total pulse charge Q_{gT} (and hence the energy of the absorbed γ-ray) for each captured event. We note that if more filters are measured than there are parameters Q, the matrix can be inverted using least squares methods to obtain **Eqn. 23b**.

4. Evaluation for two filter sets

4.1. A four-filter set with typical parameters

We now evaluate a typical case of the filter set leading to **Eqns. 22** and **23**: we set m equal to 4 μs, g to 1 μs and assume typical values of $\tau_1 = 50$ μs and $\tau_2 = 1$ ms with a division ratio $\beta = 0.04$. In this case, for 40 MHz sampling, m = 160, g = 40 and we can compute

$$\mathbf{J} = \begin{bmatrix} 1.54041E+2 & -1.54138E+2 & -8.08795E+1 & -8.00411E+1 \\ 1.98087E-2 & -1.06150E+0 & -2.00007E-3 & -1.99953E-3 \\ -2.95906E+0 & 3.07604E+0 & -1.53785E+1 & -1.59682E+1 \\ 0.00000E+0 & 0.00000E+0 & -1.69959E+1 & -1.60483E+1 \end{bmatrix} \tag{24a}$$

$$\mathbf{J}^{-1} = \begin{bmatrix} 6.309486E-3 & -9.623578E-1 & -1.593231E-2 & -1.549595E-2 \\ 1.190638E-4 & -9.600143E-1 & -2.284270E-4 & -2.469335E-4 \\ 1.194301E-2 & 6.874687E-2 & 6.221807E-1 & -6.786521E-1 \\ -1.264822E-2 & -7.280624E-2 & -6.589193E-1 & 6.564133E-1 \end{bmatrix}, \tag{24b}$$

so

and
$$Q_{gT} = 6.309486E-3\,\sigma_\Sigma - 4.811789E-4\,\sigma_g - 1.593231E-2\,\sigma_{43} - 1.549595E-2\,\sigma_{21} \tag{24c}$$

σ_g is typically of order $0.25\sigma_\Sigma$, so the σ_g term is typically less than 2% of Q_{gT}. In spite of its small size, however, σ_g carries the burden of the ballistic deficit correction, since it is the only filter that probes the step's risetime directly.

The coefficients in **Eqn. 24c** also depend upon the value β. When the pole-zero is perfectly balanced, ($\beta = 0$) **Eqn. 24c** becomes:

$$Q_{gT} = 6.309486\text{E--}3\ \sigma_\Sigma - 4.811789\text{E--}4\ \sigma_g - 1.593231\text{E--}2\ \sigma_{43} - 1.549595\text{E--}2\ \sigma_{21}, \quad (25)$$

where σ_g's coefficient has changed by 4%, σ_Σ's by 0.04%, and the σ_{43}'s and σ_{21}'s by about 0.008%. The need for β accuracy is therefore a function of how accurately one wishes to correct for risetime fluctuations and pole-zero inaccuracies. The J^{-1} weighting coefficients are found to vary linearly with β, which allows β to be determined experimentally if it is not precisely known. To do this, first select a set of β values bracketing its expected value and compute **Eqns. 24** to produce a set of weights $\{J_{1n}^{-1}(\beta)\}$ (the coefficients in **Eqn. 24C**) for each β value. Next collect data for each weight set $\{J_{1n}^{-1}(\beta)\}$ and measure the standard deviation $\sigma Q_{gT}(\beta)$ in Q_{gT} (i.e., the energy resolution) which is obtained. Then, by plotting or interpolating the $\sigma Q_{gT}(\beta)$ results versus β, select a β value which minimizes $\sigma Q_{gT}(\beta)$. If the preamplifier is stable, β should only need to be determined once.

We tested the accuracy of **Eqn. 24c**'s in the presence of both risetime variations and a sizable τ_2 decay term (equal to the τ_1 step amplitude) using a spreadsheet to compute the filter responses for modeled waveforms. **Table 1** presents these results. As may be seen, the **Eqn. 24c** is insensitive to both risetime fluctuations and τ_2 components at the few ppm level and is three orders of magnitude better than a simple trapezoidal filter.

Table 1: Modeled charge Q_{gT} for 5 filters in the presence of varying risetimes and τ_2 amplitudes for the case of β equals 0.02.

	Risetime = 450 ns		Risetime = 950 ns	
Model	$\langle\tau_2\rangle = 0$	$\langle\tau_2\rangle = 1$	$\langle\tau_2\rangle = 0$	$\langle\tau_2\rangle = 1$
Triangular	0.861966	0.859787	0.831149	0.827170
	(100.00%)	(99.75%)	(96.43%)	(95.96%)
Trapezoidal	0.867397	0.862421	0.863901	0.858928
	(100.00%)	(99.43%)	(99.59%)	(99.02%)

Section 4.1 (Eqn. 24c)	0.999997 (100.000%)	0.999997 (100.000%)	1.000001 (100.000%)	1.000001 (100.000%)
Section 4.2 (Eqn. 33)	1.000119 (100.000%)	1.000113 (99.999%)	1.000028 (99.991%)	1.000022 (99.990%)

4.2. A three-filter simplification which still has good performance

For cases which do not demand the precision of **Eqn. 24c**, or have small τ_2 components, we show a simpler, three-filter set which still has good performance. We use trapezoidal/triangular filters for their insensitivity to the "DC-like" slow variation in the τ_2 component: namely the same "energy" and "risetime probe" filters as in § **4.1** and a second trapezoidal filter σ_{32} (**Eqn. 20c**), aligned to exclude the risetime region, to estimate Q_{13}. Thus:

$$
\begin{bmatrix} -\sigma_\Sigma \\ \dfrac{\sigma_g}{r_1} \\ W\sigma_{32} \end{bmatrix} = \begin{bmatrix} -A_{1,m} & A_{1,m} & -\alpha(1,m,-m-g) \\ 0 & 1 & b_1^{-g}\dfrac{\alpha(1,g,-g)}{r_1} \\ WA_{1,\frac{m}{2}} & -WA_{1,\frac{m}{2}} & W\alpha\left(1,\frac{m}{2},\frac{-m}{2}-g\right) \end{bmatrix} \begin{bmatrix} Q_{gT} \\ Q_c \\ Q_{13} \end{bmatrix}.
$$

(26)

In **Eqn. 26** we reversed the sign of σ_Σ in order to create a filter that is differential on both sides of the step's rising edge and we weighted σ_{32} by W to determine the value that gives the best results. We find the lowest noise is for $W = 1$, so we can repeat **Eqns. 24a** and **24b** to obtain:

$$
Q_{gT} = 9.338925E{-}3\,\sigma_\Sigma - 5.000000E{-}4\,\sigma_g + 3.106015E{-}2\,\sigma_{32}.
$$

(27)

Table 1 shows that this result is sensitive to the τ_2 component at the few ppm level and to risetime variations at the 100 ppm level. For many practical cases, this is sufficiently accurate. Assuming a random distribution of risetimes, for example, this would add less than 100 eV in quadrature to the energy resolution at 1 MeV, which would be scarcely perceptible.

5. Correlated multiple output sampling in real time implementations

5.1. Defining "correlated multiple output sampling"

The filter weighting **Eqns. 25** and **27** derivations were developed as if the signals had been captured into memory, so the running-sums could simply be taken over the desired sample values. This implementation is feasible at lower data rates, but in the

preferred implementation, as taught by WARBURTON-1997, a gate array is used to realize the difference of running-sum filters, which run in real time, each outputting a new value its own fixed number of clock cycles after a new ADC sample is supplied. A priori, then, there may be no single instant when a set of filter values can be captured which will

5   reproduce the inter-filter time relationships required by **Eqns. 25** and **27**. Instead, in general, the outputs of the different filters will have to be captured at different times and these times will have to be carefully correlated if **Eqns. 25** and **27** are to produce accurate values of Q_gT. Because multiple filter values must be captured, we call the resultant set of captured values a "correlated multiple output sample" or "cMOS" and refer to these

10   values either as "cMOS values" or as "individual sample values in the cMOS".

We also observe that the number of cMOS values required by **Eqns. 25** and **27** and number of realized filters needed to produce them need not be the same. For example, in **Eqn. 25**, the σ_{21} and σ_{43} filter values both come from triangular filters of length m/2 with their starting points separated by the interval m+g. Thus two outputs

15   from the same realized filter can be captured to supply the two required values.

While we have noted that the filter ranges do not have to be precisely positioned *relative to the step's rising edge*, the accuracy of their positions *relative to one other* in the cMOS capture determines the accuracy that the method can attain. This is because the running-sum ranges, which translate directly into filter lengths and time

20   intervals between starting locations, are encoded into the J^{-1} weighting coefficients through terms like $\alpha(j,r,s) = A_{j,r}(1-b_j^s)$ (see, e.g., **Eqn. 6**), where r is the filter length and s the interval between the sum start points. For simplicity, we aligned the various filters so their starting points coincided, but other cases are trivially treated by scaling **Eqns. 4-7** by terms like b_j^t, where t is the offset between starting locations. Therefore, there is an

25   intimate relationship between the weighting coefficients and the filter ranges which, if violated, will introduce inaccuracies. The J^{-1}'s are computed for a particular set of filter sum ranges. If another set of ranges is captured, then expected cancellations between filter terms will not occur exactly and the differences will appear as errors in the detected charge. In digital implementations, the fact that the signal samples are discrete and

30   enumerated makes is relatively easy to maintain accurate time relationships by measuring time in "ticks" of the digitization clock and then simply counting out the required time intervals.

The requirement of appropriate time correlations between the cMOS values clearly differentiates the present method from prior art as represented by the

35   approaches of HINSHAW-1991 and KUMAZAWA-1988. While two filter outputs were captured and weighted in those methods, the capturing steps used peak detecting means. The methods are therefore amplitude based rather than time based, as in the present method, and the captured values carry different information about the step shapes.

Further, the method of capturing based on peak detection assures that there will be no fixed time relationship between the captured samples since the two filters' times of maximum amplitude depend differently on step shape, which will vary from step to step.

In light of the above, it is clear that there will be a wide variety of possible
5   approaches to capturing filter cMOS values with the prescribed time relationships. The outputs of single filters can be captured multiple times. Filter outputs can be captured as they appear from the gate array or can be delayed for convenience. We will show several examples in § 6. Which method is best will depend upon implementation details and will vary from case to case. We therefore intend the phrase "cMOS" (for "correlated multiple
10   output sample") to cover the multiplicity of possibilities for capturing, in a real-time implementation, filter values whose time relationships are correct for the set of J^{-1} weighting coefficients that will be applied to them to recover the input charge from the detected event. When the cMOS capture is made in response to the detection of a step in the preamplifier's output signal, with the intention of using it to determine the charge
15   deposited in the detector (in the general case the impulse area) and hence the gamma-ray's energy, we will often refer to it as an "energy cMOS capture" (or "charge cMOS capture" or "area cMOS capture" as appropriate).

5.2. Timing considerations

Filters, in real time implementations, require finite times to compute. For
20   example, consider a trapezoidal filter of length (risetime) m and gap g over a set of values $\{q_i\}$, where q_n is the last sample value ("end sample") to enter the filter:

$$\sigma_\Sigma(n) = \sum_{i=n-m}^{n} q_i - \sum_{i=n-2m-g}^{n-m-g} q_i \qquad (28)$$

If implemented in gate arrays according to the method of WARBURTON-1997, the output $\sigma_\Sigma(n)$ will appear 3 filter clock cycles after sample q_n is presented. This is its
25   propagation delay. For a 4 μs peaking time (filter clock equals 200 ns), the propagation delay is 600 ns. These delays must be included in the design to assure that the cMOS capture constraints are satisfied.

Figs. 5A and 5B show how to compute sampling times for the Eqn. 24c filter set. The calculation has two steps: finding the time delays between their end
30   samples of the different filters and then correcting for their different propagation delays. For the first step we use Fig. 5A, which shows the filter regions relative to the preamplifier output step 72. σ_Σ 74, σ_{43} 76, σ_{21} 77, and σ_g 79 were all described earlier. σ_t 81 is a fast channel timing filter, as described by WARBURTON-1997, which is used to detect step-like signals in the preamplifier's output. The timing filter's output triggers
35   a digital discriminator when it is located approximately as shown with respect to the leading edge of the signal step 72. We can now measure end sample delays relative to σ_t

81 as shown, with the filters σ_Σ 74, σ_{43} 76, and σ_g 79 appearing at times Δt_Σ, Δt_{43}, and Δt_Σ after σ_t 81 and σ_{21} 77 appearing Δt_{21} earlier. From **Fig. 5A**, if σ_t has a peaking time of T, then Δt_Σ equals (m+g-2T).

It is important to note that, while the firing of σ_t may jitter somewhat depending upon the step's risetime shape, this only causes the step location to jitter within the gap g. The relative timings of all the filters to σ_t, and thus to each other, are fixed by construction. Thus, provided σ_g is wide enough to catch the step for any reasonable discriminator triggering, the filter will work as designed. This relative insensitivity to the timing accuracy with which the step-like signals are detected is a significant advantage of the present invention.

Fig. 5B shows the second step: correcting for propagation delays. We note that these delays may exceed a microsecond in long time constant filters and typically increase with filter length. **Fig. 5B** shows propagation delays with respect to the ADC sample time t_s 83 for a 40 MHz implementation with g = 1 μs and m = 4 μs, so δt_t = 75 ns, $\delta t_g = \delta t_{21} = \delta t_{43}$ = 300 ns, and δt_Σ = 1,200 ns. Combining **Figs. 5A and 5B**, we should sample the output of a given filter σ_x, at

$$t_x = \Delta t_x + \delta t_x - \delta t_t \tag{29}$$

following the discriminator firing on σ_t's output to assure the cMOS temporal relationships required to extract $Q_g'T$. If t_x is positive, then a counter can be used to measure t_x and capture the filter value. If t_x is negative, however, then its propagation delay δt_x will have to be increased by at least $|t_x|$ to meet the demands of causality (capturing must follow step detection in time). In digital implementations this is a simple operation using a FIFO memory. In our present example, no additional delay is required since a typical timing filter peaking time is 100 ns, so Δt_{21} = -200 ns and

$$t_{21} = \Delta t_{21} + \delta t_{21} + \delta t_t = -200 + 300 - 75 = 25 \text{ ns}, \tag{30}$$

which is positive.

6. Circuits implementing the cMOS method

6.1. Fully digital filter implementations

Two general approaches are possible: 1) asynchronously capturing values as they appear from the sub-filters, per the discussion of **Sect. 5.2**, and 2) extending the filters' propagation delays so that they can be synchronously captured. In the latter case all pileup inspection is carried out before the capture signal is given, while in the former case the values are captured as they appear and the pileup inspection is used to decide whether anything should be done with them afterwards. We will demonstrate one implementation of each to show the principles.

6.1.1. Asynchronous capture implementation

Fig. 6A shows a preferred implementation of **Eqn. 24c**. The digital processor **90** comprises an ADC **92**, a real-time digital processing unit (RT-DPU) **93**, and a digital signal processor (DSP) **95**. Per the method taught by WARBURTON-1997, the RT-DPU processes data at the ADC output rate while the DSP processes data at the event rate (impulse signal rate). The RT-DPU produces values σ_Σ, σ_{43}, σ_{21}, and σ_g, while the DSP multiplies them by the coefficients J^{-1} to obtain Q_{gT}.

The RT-DPU **93** also has two parts, a fast channel **97** and a number of slow channels **98**. The fast channel contains the fast timing filter σ_t **100**, a digital discriminator **102**, some pileup test and timer logic **103**, a clock **104**, and an output register **105**. Each slow channel comprises a triangular or trapezoidal digital filter and an output register with either a digital delay line (if **Eqn. 29** is negative) or a timer (if **Eqn. 29** is positive). Notice that σ_{43} and σ_{21}, both being of length m/2, are being implemented using a single triangular filter σ_{4321} **107**.

In operation, σ_t **100** processes ADC **92** data until a step-like signal arrives. Discriminator **102** detects this step and signals the pileup inspector **103**, which strobes a trigger line **106** and initiates pileup inspection. If the step's arrival time is important, the output of clock **104** can be captured to output register **105** by the same strobe signal **106**. Filter σ_{4321}'s **107** output, delayed by t_{21} using 21-delay **108** if required by **Eqn. 29** causality, is captured immediately in the 21-register **109** by this trigger strobe, which also starts the 43-timer **110**, Σ-timer **111**, and g-timer **112** to time t_{43}, t_Σ and t_g. Since t_g is shortest, the g-timer **112** times out first, capturing filter σ_g's **115** output in the g-register **116**. Similarly, the 43-timer **110** and Σ-timer **111** time out at t_{43} and t_Σ to capture filter σ_{4321} **107** and σ_Σ **117** outputs in the 43-register **118** and Σ-register **119**. If this step is not piled up, then all four filter values are ready to be read into the DSP **95** via data bus **121**. The pileup inspector can be implemented as taught by WARBURTON-1999 relative to the length of the σ_Σ filter and will not be discussed further.

As per the cMOS discussion in § **5.1**, the invention method does specifically require that the number of filters equal the number of filter values to be captured. Here we have captured σ_{4321} twice.

6.1.2. Synchronous capture implementation

Fig. 6B shows an implementation of **Eqn. 27** wherein the three filter values are captured just as the value emerges from the slowest (σ_Σ) filter. The primary physical difference between this implementation and that of **Fig. 6A** is that the timers **108, 109,** and **110** have been eliminated and a second delay (g-delay **125**) has been added to compensate for the differences between t_Σ and t_g and t_{32} from **Eqn. 29**. All the remaining parts are the same and carry the same reference numbers as is **Fig. 6A**. Notice that the trigger line **106** now connects directly to the output registers, rather than through

timers as before. For this topology, the functional operation of the pileup test and timer #1 **103** is essentially identical to that presented by WARBURTON-1999.

A disadvantage of this circuit, compared to **Fig. 6A**, is that relatively long delays may be required, which may consume excessive gate array resources. Recent
5   programmable gates, however, are appearing with blocks of dedicated memory, so this restriction is becoming less significant. One significant advantage of this design is that, because of the synchronous capture, the pileup inspection logic design is much simpler.

6.2. Fully analog implementation

While our preferred implementation is fully digital, the method is not
10   inherently digital *per se*. In certain cases, particularly for very fast step risetimes and very short filter time constants, or to allow a high degree of integration, it may become preferable to use an analog implementation. **Fig. 7** shows an analog version of **Fig. 6A**. The topologies are identical up to the point where sample and hold circuits replace digital registers to capture the filter values, so the operation of the circuit up to this point will be
15   clear to one skilled in the art, given the teaching presented herein. The derivation of the J matrix, of course, will have to proceed using weighting functions appropriate for the analog filters used. Then analog multipliers (**160**, etc.) and an analog adder **185** are used to implement the J^{-1} coefficients and addition of **Eqn. 24C**. We note that, for an application with fixed filters, the multiply-and-add function is just an op-amp circuit with
20   appropriate resistor inputs to its summing node. If the pileup tester **143** deems the value good, the gate and shape circuit **145** passes out the adder **185** output in a form suitable for multichannel analysis.

6.3. Hybrid implementations

Shaping filter characteristics which are difficult to implement digitally
25   may be required in some situations where the other benefits of digital processing are still desired. In these situations hybrid implementations may be considered. **Fig. 8** shows one such implementation **190**. The topology is nearly identical to that of **Fig. 6A**. The major differences are: 1) that the fast channel **191** is implemented in analog circuitry identical to that of **Fig. 7** (although the digital circuit of **Fig. 6A** could as easily be used); and 2) that,
30   in the slow channel section **192**, the digital filters σ_{4321} **107**, σ_Σ **117**, and σ_g **115** have been replaced by analog filters σ_{4321} **193**, σ_Σ **194**, and σ_g **195** followed by ADCs **196**, **197**, and **198**. The time behavior of this circuit is the same as was described for the asynchronous digital circuit in **Fig. 6A**.

7. Noise considerations

The **Eqns. 24C** and **27** filters have higher noise than simple trapezoidal filters since they weight the sampled set of data points with values larger than unity. For example, by breaking the 2m+g filtered region of **Eqn. 27** into its 4 major sub-regions of length m/2 (ignoring the gap g for this argument) and calling running-sums over these regions σ_1 through σ_4, we can recompose **Eqn. 27** as:

$$Q_{gT} \cong -9.338925E{-}3\ \sigma_4 + 2.172123E{-}2\ \sigma_3 - 2.172123E{-}2\ \sigma_2 + 9.338925E{-}3\ \sigma_1 \quad (31)$$

If the noise standard deviation per sample is σ, then the total noise $\mathrm{sig}(Q_{gT})$ will be

$$\mathrm{sig}\left(Q_{gT}\right) = \left(160\left(8.722E{-}5\ \sigma^2\right) + 160\left(4.718E{-}4\ \sigma^2\right)\right)^{0.5} = 0.299\ \sigma \quad (32)$$

which can be compared to a trapezoidal filter of the same length:

$$\mathrm{sig}(Q) = \left(160\left(\frac{\sigma}{160}\right)^2 + 160\left(\frac{\sigma}{160}\right)^2\right)^{0.5} = 0.112\ \sigma \quad (33)$$

Thus the **Eqn. 27** filter will have approximately 2.7 times as much electronic noise as a simple trapezoidal filter. The result for the **Eqn. 24c** filter is similar. For a 4 μs filter, whose electronic noise can be as low as 150 eV, this effect is significant at energies like 100 keV (Fano noise 415 eV) but not at 1 MeV (Fano noise 1,300 eV). And, as peaking times shorten to obtain higher count rates, the problem will worsen at higher energies. We therefore consider how to obtain the benefits of correcting for both ballistic deficit and pole-zero errors without paying the additional filter noise price.

8. Treating non-ideal terms with baseline measurements

Our approach is to extend the baseline averaging concept of WARBURTON-1997 to treat time varying terms as well. In the particular case at hand, we would like a way to measure Q_{23} using baseline measurements in spite of the fact that it both exhibits exponential decay and is increased each time an event occurs.

8.1. Derivation of an example filter

We begin by developing a simple cMOS filter, composed of three running averages, of lengths m, g, and m, which we make contiguous for the practical reasons of simplifying the math and minimizing deadtime, but not for any fundamental reason. As in previous sections, the length m filters will primarily serve to determine Q_{gT}, while the length g filter will be used to eliminate ballistic deficit effects by probing the signal step's risetime region. Starting with **Eqns. 19A, 19B,** and **19D**, these filters are:

$$\sigma_{\Sigma+} = Q_{gT}\overline{A}_m - Q_c\hat{A}_m + Q_{13}A_{1,m} \quad + Q_{23}A_{2,m} \quad + mB_0, \quad (34a)$$

$$\sigma_{g-} = \qquad Q_c \dot{r} \quad + Q_{13}b_1^{-g}A_{1,g} \quad + Q_{23}b_2^{-g}A_{2,g} \quad + gB_0 \tag{34b}$$

and $\qquad \sigma_{\Sigma-} = \qquad\qquad + Q_{13}b_1^{-m-g}A_{1,m} + Q_{23}b_2^{-m-g}A_{2,m} + mB_0$ (34c)

where

$$\overline{A}_m = (1-\beta)A_{1,m} + \beta A_{2,m}, \quad \widehat{A}_m = A_{1,m} + FA_{2,m}, \quad \dot{r} = r_1 + Fr_2 \tag{34d}$$

5   In **Eqns. 34** we now have 4 classes P_i of parameters: Classes 1 and 2 (P_1, P_2), as before, which include Q_{gT}, Q_c, and Q_{13}; Class 3 (P_3), Q_{23} and B_0, which describe the preamplifier's "non-ideal" behavior, Q_{23} being the amplitude of its second pole's response to any previous steps and B_0 being the DC offset; and Class 4 ($P4$): all the remaining fixed parameters (e.g., τ_1, τ_2, β, m, g) which describe both the filters' and the

10   preamplifier's transfer functions. We will treat Q_{gT}, Q_c, and Q_{13} as unknowns to be recovered from a particular cMOS filter measurement, with the intention of independently determining Q_{23} and B_0 from baseline measurements. **Eqns. 34** are simple enough to be solved by direct elimination of unknowns to get:

$$Q_{gT} = \overline{A}_m^{-1}\left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\widehat{A}_m}{\dot{r}}\sigma_{g-}\right]_q - \overline{A}_m^{-1}\left[LQ_{23} + MB_0\right] \tag{35a}$$

15   where

$$K = b_1^{m+g} + \frac{\widehat{A}_m}{\dot{r}}\frac{A_{1,g}}{A_{1,m}}b_1^m \tag{35b}$$

$$L = A_{2,m}\left[1 - \left(\frac{b_1}{b_2}\right)^{m+g} + \frac{\widehat{A}_m A_{2,g}}{A_{2,m}\dot{r}}b_2^{-g} - \frac{\widehat{A}_m A_{1,g}A_{2,m}}{A_{1,m}\dot{r}}b_1^{-g}\left(\frac{b_1}{b_2}\right)^{m+g}\right] \tag{35c}$$

and

$$M = m\left[1 - b_1^{m+g} + \frac{\widehat{A}_m}{\dot{r}}\left[\frac{g}{m} - \frac{A_{1,g}}{A_{1,m}}b_1^m\right]\right] \tag{35d}$$

20   The constants K, L, and M depend only on Class 4 parameters, which are assumed to be time independent for a particular detector-preamplifier pair and set of filter lengths. **Eqn. 35A** is therefore a transform function (the first bracketed term) between the **Eqn. 34** cMOS filter values $\sigma_{\Sigma+}$, $\sigma_{\Sigma-}$, and σ_{g-} and the desired detector charge Q_{gT}, minus an error term $[LQ_{gT} + MB_0]\overline{A}_m^{-1}$, that is the transform's response to the non-ideal (NI)

25   preamplifier terms Q_{23} and B_0. Using **Eqn. 35a** thus requires that Q_{23} and B_0 to be determined at a time appropriately correlated to the cMOS capture (e.g., time 1 in **Fig. 4**) and substituted into the error term to produce an accurate value of Q_{gT}. The cMOS

capture itself can be physically implemented using any of the approaches described above. The subscript q on the bracketed weighted sum of cMOS values indicates that this was an "area" cMOS capture of the area filter set to determine deposited charge.

8.2. Single measurement error term correction

A simple, although not the lowest noise, method to estimate the error term is to measure it locally for each cMOS energy capture by making the same **Eqn. 34** cMOS filter capture in close time proximity but without a step-like signal present in the filters, so that Q_{gT} is zero. Then, from **Eqn. 35a**, with Q_{gT} equal to zero, we have:

$$\overline{A}_m^{-1}\left[LQ_{23} + MB_0\right]_b = \overline{A}_m^{-1}\left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\hat{A}_m}{\hat{r}}\sigma_{g-}\right]_b , \qquad (36)$$

where the subscript b is for "baseline", called so since, as shown, this "baseline" cMOS filter capture produces the transform's baseline response to the preamplifier's non-ideal terms. We may also call such a measurement a "null charge" or "null energy" ("null area" in the general case) capture since it is made when QgT is zero. Substituting **Eqn. 34** c into **Eqn. 33A** gives:

$$Q_{gT} = \overline{A}_m^{-1}\left[\left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\hat{A}_m}{\hat{r}}\sigma_{g-}\right]_q - \left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\hat{A}_m}{\hat{r}}\sigma_{g-}\right]_b\right] , \qquad (37)$$

where the subscript q labels the cMOS charge measurement to be corrected.

This correction is moderately simple and has been found to work acceptably in some cases. However, if Q_{23} is large, as may be true at high output step rates, then time decay effects in $[LQ_{23} + MB_0]_b$ may still be significant. This can be approximately corrected by measuring the time interval $\delta t = t_e - t_b$ (in clock ticks Δt) between the energy and baseline captures and correcting by the decay term $b_2^{\delta t}$, to obtain:

$$Q_{gT} = \overline{A}_m^{-1}\left[\left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\hat{A}_m}{\hat{r}}\sigma_{g-}\right]_q - b_2^{\delta t}\left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\hat{A}_m}{\hat{r}}\sigma_{g-}\right]_b\right] . \qquad (38)$$

In this method, if B_0 is truly a constant and the time interval δt between the energy and baseline captures is fixed, then the error term $MB_0 b_2^{\delta \tau}$, will also be a constant and merely produce an offset in the spectrum.

Eqn. 38 has two significant problems: 1) increased noise, due to the noise in the single baseline measurement, which is the same size as in the measurement itself and degrades resolution by sqrt(2); and 2) it increases deadtime by at least a factor of two if the two measurements are independent and so separated by at least 2m+g in time. It is

therefore useful to be able to make multiple, independent measurements of B_0 and Q_{23} and average them in order to reduce their variance. We present a method for doing so below.

8.3. The time compensated baseline model

5   In making filter derivations throughout this work, we have assumed that we have an accurate preamplifier model in terms of its several decay constants and DC offset. We therefore take the logical step and use precisely the same model to correlate measurements made at different times so they may be averaged. Thus, if we measure Q_{13}, Q_{23}, or B_0 at one time, then their values Δt clock cycles later will be given by

10      $b_1^{\Delta t}Q_{13}$, $b_2^{\Delta t}Q_{23}$, and B_0, respectively, unless a step arrives within the interval Δt. In that case, at the end of the step arrival time (e.g., time 1 in **Fig. 4**), we increment Q_{13} by $(1-\beta)Q_{gT}$, Q_{23} by βQ_{gT}, and B_0 by zero, after which their exponential decays will continue as before. For the purposes of baseline averaging, then, Q_{13}, Q_{23}, or B_0 become "time-compensated" model (or "TC-model") parameters, whose averaged values are $<Q_{13}>$,

15      $<Q_{23}>$, and $<B_0>$, respectively.

If we make measurements of B_0 and Q_{23} from time to time, we can now update our estimate of B_0 simply by using the method of WARBURTON-1997 or some other averaging scheme. For Q_{23} our TC-model now gives:

$$Q_{23}(i) = b_2^{\Delta t} Q_{23}(i-1) \, , \tag{39}$$

20   so, if $<Q_{23}>_{i-1}$ represents an "averaged" estimate of $Q_{23}(i-1)$, last measured at time t_{i-1}, we update $<Q_{23}>_i$, using a new baseline measurement of $Q_{23}(i)$ at time t_i, by the following extension of Warburton and Hubbard's **Eqn. 7**, where $\Delta t_i = t_i - t_{i-1}$:

$$\langle Q_{23}\rangle_i = \frac{N-1}{N}\left(b_2^{\Delta t_i}\langle Q_{23}\rangle_{(i-1)}\right) + \frac{Q_{23}(i)}{N} \, . \tag{40}$$

Equivalently, we update $<Q_{23}>$ for the arrival of an impulse signal of charge $Q_{gT}(i)$ at

25   time t_i using:

$$\langle Q_{23}\rangle_i = b_2^{\Delta t_i}\langle Q_{23}\rangle_{(i-1)} + \beta Q_{gT}(i) \, . \tag{41}$$

If required, Q_{13} can be similarly extended from its last time of measurement by replacing b_2 with b_1 in **Eqn. 39**.

$<Q_{23}>$ and $<B_0>$ are used with an charge cMOS capture to determine Q_{gT}

30   using **Eqn. 35a** as follows. First, $<B_0>$ simply replaces B_0 in the equation. $<Q_{23}>$, however, has to be extended from its time of last update t_{i-1} to the present capture time t_i using **Eqn. 39**, according to our TC-model, where $\Delta t_i = t_i - t_{i-1}$:

$$Q_{gT} = \overline{A}_m^{-1} \left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\hat{A}_m}{\bar{r}}\sigma_{g-} \right]_q - \overline{A}_m^{-1} \left[Lb_2^{\Delta t_i} \left\langle Q_{23} \right\rangle_{i-1} + M\left\langle B_0 \right\rangle \right]$$

(42)

Because <Q23> and <B0> can be determined by averaging over many baseline measurements, the noise contribution of the correction term in **Eqn. 42** can be made much smaller than in the single baseline correction procedure of § **8.2**. Since the

5   weighting function of σ_{g-} is small compared to unity, the measurement noise in **Eqn. 42** will then be determined primarily by $\sigma_{\Sigma+}$ and $\sigma_{\Sigma-}$, and will be approximately the same as for a trapezoidal filter of the same peaking time and about a factor of sqrt(2) better than by using **Eqn. 38**. Furthermore, because a baseline cMOS filter capture can now be made at any time the interval between successive steps is adequately long, rather than requiring

10   one in conjunction with each charge cMOS filter capture, the method's deadtime is cut in half: to that of a single cMOS filter capture.

8.4. Measuring the non-ideal terms Q23 and B0

An effective way to measure Q23 at time t_i is by making a baseline cMOS capture, per **Eqn. 36**, with the difference now that the times t_i need no longer be

15   correlated with charge cMOS captures. Using an averaged value <B0> for B0, plus computed values for L and M, we can then solve [LQ23 + MB0]b for Q23:

$$Q_{23} = \frac{1}{L} \left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\hat{A}_m}{\bar{r}}\sigma_{g-} \right]_b - \frac{M}{L}B_0$$

(43)

and use it to update <Q23>i using **Eqn. 40**.

We normally expect B0 to be stable and nominally rate independent, so

20   that <B0> can be measured with no input counts to the detector and used thereafter to obtain Q23 values. However, to compensate for any temperature and/or rate related changes in B0, it is prudent to also measure B0 from time to time and update our estimate of <B0>. One approach is to use pairs of baseline measurements, separated in time by Δt, where, if we denote the error term E as:

25

$$E = \left[LQ_{23} + MB_0 \right]_b = \left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\hat{A}_m}{\bar{r}}\sigma_{g-} \right]_b$$

(44)

then, measuring E_i and E_{i-1} separated by Δt = t_i - t_{i-1}, we have, from **Eqn. 39**:

$$B_{0,i} = \left(E_i - b_2^{\Delta t_i} E_{(i-1)} \right) \left(M \left(1 - b_2^{\Delta t_i} \right) \right)^{-1}. \tag{45}$$

$B_{0,i}$ measurements can be averaged, as noted above, to obtain $<B_0>$, a low variance estimation of B_0, for use in **Eqn. 35a**. From **Eqn. 45**, it is clear that the longer the time interval Δt, the more accurate the measurement of B_0 will be, since both the numerator and denominator are small differences of large numbers. In any case, enough values $B_{0,i}$ can be averaged so that errors in $<B_0>$ do not contribute substantially to the error in Q_{gT}.

8.5. Dealing with model parameter drifts

For this compensation scheme to work well, the P_4 parameter β needs to be precisely known and stable over time. Else both the extrapolations of Q_{13} and increments in Q_{23} will incorporate systematic errors which will propagate into the Q_{gT} calculations. The proposed Q_{23} baseline measurements offer a method to monitor β and either refine its value or track it in time.

In **Eqn. 40** we update our average $<Q_{23}>_i = b_2^{\Delta t}<Q_{23}>_{i-1}$ at time t_i with the measured value $Q_{23}(i)$. However, we can also record the differences between these two values and compute their mean $<\Delta Q_{23}>$ over time. If $<\Delta Q_{23}>$ is not zero, then a systematic error exists and we can adjust β using:

$$\delta\beta = <\Delta Q_{23}>/S_\beta, \tag{46}$$

where the slope $S_\beta = d<\Delta Q_{23}>/d\beta$ is previously obtained from calibration runs, made at system setup time, wherein β is systematically adjusted and the resultant $<\Delta Q_{23}>$ values recorded, so that S_β can be calculated. Similar methods can be used to monitor and correct other P_4 parameters such as τ_1 and τ_2, if appropriate.

9. References

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35

10. Conclusion

The foregoing description of specific embodiments has been presented for purposes of illustration and description. It is not intended to be exhaustive or to limit the invention to the precise forms described, and obviously, many modifications and
5   variations are possible in light of the above teaching. These embodiments were chosen and described in order to best explain the principles of the invention and its practical application to thereby enable others in the art to best utilize the invention in various embodiments and with such modifications as best suit the invention to the particular uses contemplated.

10   As a first example, while all the described embodiments used trapezoidal or triangular filters, there is no such restriction in the technique. As shown, any filter describable by a set of weighting constants applied to successive signal samples, or its analog equivalent, can be used. The method will therefore work with essentially any realizable filter. Second, while the specific embodiments employed two, three, or four
15   filters, there is no specific limit on the number of filters which can be used. Even a single filter can be used if its output is correctly sampled at several times. If the number of filters N exceeds the number of base amplitudes M, then matrix inversion methods can be used to find least squares best estimates of the M values, rather than exact values. Third, while all the filters used in the specific embodiments were symmetric, this is not a
20   requirement of the method, as may be seen from the derivations. Fourth, while specific configurations of electronic circuits were shown in the implementations, other configurations can readily be devised which accomplish the same functions. The number of ADC bits or value lengths captured to the registers could be varied, for example. Fifth, even the use of electronic circuits is not required to implement the method. As per the
25   initial derivations, for example, if successive ADC values are instead read into computer memory, all the subsequent processing steps can be carried out in software. Sixth, the method can also be applied to signals which are formally equivalent to outputs from nominally single pole filters, whether the signal source is actually a single pole filter or not. Superconducting bolometers produce such output signals, for example.

30   Therefore, the above description should not be taken as limiting the scope of the invention, as defined by the appended claims.

<u>WHAT IS CLAIMED IS:</u>

1   1. A method for determining the integrated area Q_gT of a pulse-like input
2   signal by measuring a step-like output signal provided by a nominally single-pole (N-1P)
3   device in response to said pulse-like input signal, the method comprising:
4   applying a filter set having one or more filters to said N-1P device output;
5   detecting the presence of a step-like feature in said output signal;
6   in response to detecting said feature, capturing a set of correlated multiple
7   output sample values (the area cMOS) from one or more filters in said filter set; and
8   forming a weighted sum of the sample values in said area cMOS to
9   determine said integrated area Q_gT (the determined area) of said input signal;
10   wherein the weights in said sum (the area weights) are selected to
11   compensate said determined area for errors arising either from the time structure of said
12   pulse-like input, or from deviations in the N-1P device's response from an ideal
13   single-pole response, or from both.

1   2. The method of claim 1 wherein said pulse-like input signal has a finite
2   time extent and a possibly time-varying amplitude as well.

1   3. The method of claim 2 wherein:.
2   both DC and long decay constant terms in said N-1P device's output signal
3   are ignored in computing the weights used in determining the integrated area Q_gT ;
4   a longer time constant trapezoidal filter is used primarily to estimate the
5   amplitude of said step-like feature;
6   an intermediate time constant trapezoidal filter is used to estimate the
7   device's first-pole response to prior pulse-like inputs; and
8   a shorter time constant triangular filter is used to measure the risetime
9   characteristics of said step-like feature.

1   4. The method of claim 2 wherein:.
2   both DC and long decay constant terms in said N-1P device's output are
3   ignored in computing the weights used in determining the integrated area Q_gT ;
4   two longer time constant running average filters are used primarily to
5   estimate both the amplitude of said step-like feature and the device's first-pole response
6   to prior pulse-like inputs; and
7   a shorter time constant running average filter is used to measure the
8   risetime characteristics of said step-like feature.

1   5. The method of claim 1 wherein said N-1P device is not ideal, but has
2   additional poles and/or zeroes in its transfer function and possibly a DC offset as well.

1   6. The method of claim 1 wherein the number of individual sample values
2   in said area cMOS exceeds the number of filters in said filter set.

1   7. The method of claim 1 wherein the number of individual sample values
2   in said area cMOS is equal to the number of filters in said filter set.

1   8. The method of claim 1 wherein the output of said N-1P device is
2   digitized by an analog-to-digital converter, portions of the digitized output of which are
3   transferred to a memory, and at least one of said applying a filter set, detecting the
4   presence, capturing, and forming a weighted sum is carried out by a computer operating
5   under software control.

1   9. The method of claim 1 wherein said area weights are determined
2   analytically by:
3   developing a mathematical model of said N-1P device's output signal in
4   response to said pulse-like input signal in terms of:
5   i) a first (P_1) parameter which is the desired integrated area Q_{gT},
6   ii) a set of one or more second (P_2) parameters which describe said N-1P
7   device's residual amplitude response to any previous pulse-like inputs and/or its response
8   to the finite arrival time of the present pulse-like input, the number of said P_1 and P_2
9   parameters being less than or equal to the number of sample values captured in said area
10 cMOS, and
11   iii) a set of one or more third (P_3) parameters which describe the transfer
12 functions of the said N-1P device and the filters in said filter set;
13   describing by its weighting function each filter from which a value is
14 captured at least once in said area cMOS;
15   applying said weighting function to said parameterized output model to
16 produce a linear equation between each of the values captured in said area cMOS from
17 said filter and said P_1 and P_2 parameters; and
18   solving the resultant set of linear equations to obtain coefficients for
19 expressing said P_1 and P_2 parameters in terms of the sample values of said area cMOS.

1   10. The method of claim 9 wherein, if a P_3 parameter value is not well
2   known, determining its value by:

3   selecting a set of values for the P_3 parameter which bracket its expected

4   value;

5   for each member of said set of bracketing values, carrying out the method

6   of claim 1.6 to produce a set of coefficients expressing said P_1 and P_2 parameters in

7   terms of said captured sample values ;

8   using said set of coefficients to collect data to measure the value of said P_1

9   parameter and determine the uncertainty in said measurement;

10   using said uncertainties determined for the members of said set of

11   bracketing values to plot or otherwise parameterize said uncertainty in terms of said

12   poorly known P_3 parameter; and

13   using said plot or parameterization to select the value of said poorly known

14   P_3 parameter which minimizes the measurement uncertainty in said P_1 parameter.

1   11. The method of claim 1 wherein, if capturing said area cMOS would

2   violate causality for any filter in said filter set, relative to detecting the presence of said

3   step-like feature, increasing said filter's propagation delay sufficiently so that causality is

4   not violated by said capturing.

1   12. The method of claim 1 wherein timers are used to capture values from

2   said filter outputs as they appear so that capturing an area cMOS can be carried out

3   asynchronously.

1   13. The method of claim 1 wherein additional propagation delays are

2   added to said filters so that capturing an area cMOS can be carried out synchronously.

1   14. The method of claim 1 wherein said N-1P device output is digitized

2   by an analog-to-digital converter;

3   the filters in said filter set are digital filters;

4   the outputs of said digital filters are captured into registers; and

5   the determination of said integrated area Q_{gT} is carried out by a digital

6   computing device that reads the values captured in said registers, multiplies each such

7   value by a weighting coefficient, and adds them together.

1   15. The method of claim 14 wherein said digital filters are either

2   triangular or trapezoidal filters.

1   16. The method of claim 14 wherein:.

2   a cMOS value σ_Σ, captured from a trapezoidal filter, is used primarily to

3   estimate the integrated area Q_{gT};

4   a cMOS value σ_{32}, captured from a triangular filter, is used to estimate the

5   N-1P device's first pole residual responses to previous pulse-like inputs; and

6   a cMOS value σ_g, captured from a triangular filter, is used to probe the

7   risetime region of said step-like feature; and

8   said integrated area Q_{gT} is determined according to

$$Q_{gT} = J_{11}^{-1}\sigma_\Sigma + J_{12}^{-1}\frac{\sigma_g}{\tau_1} + J_{13}^{-1}\sigma_{32}$$

9

1   17. The method of claim 14 wherein:

2   a cMOS value σ_Σ, captured from a trapezoidal filter, is used primarily to

3   estimate the integrated area Q_{gT};

4   a cMOS value σ_{12}, captured from a triangular filter, is used to estimate the

5   N-1P device's first pole residual response to previous pulse-like inputs;

6   a cMOS value σ_{34}, captured from a triangular filter, is used to estimate the

7   N-1P device's second pole residual response to previous pulse-like inputs; and

8   a cMOS value σ_g, captured from a triangular filter, is used to probe the

9   risetime region of said step-like feature; and

10   said integrated area Q_{gT} is determined according to

$$Q_{gT} = J_{11}^{-1}\sigma_\Sigma + J_{12}^{-1}\frac{\sigma_g}{\tau_1} + J_{13}^{-1}\sigma_{43} + J_{14}^{-1}\sigma_{12}$$

11

1   18. The method of claim 14 wherein one or more of said digital filters is a

2   running average filter.

1   19. The method of claim 14 wherein digital timers are used to measure the

2   time intervals between said detecting a step-like feature and said capture of the individual

3   sample values in the area cMOS.

1   20. The method of claim 1 wherein the need for timing precision in

2   capturing said area cMOS relative to the position of the risetime region of said step-like

3   output signal is reduced by:

4   designating a risetime probe filter whose time constant is significantly

5   longer than the longest expected risetime;

6   adjusting the area cMOS capture so that the risetime regions of the N-1P

7   device's step-like outputs will fall entirely within it; and

8   for all other filter values that are captured, adjusting the area cMOS

9   capture so that the weighting constants their respective filters are negligible within said

10   risetime region.

1   21. The method of claim 20 wherein if one of said other filter values is
2   captured from a trapezoidal filter, adjusting the area cMOS capture so that the risetime
3   region lies within its gap and if one of said other filter values is captured from a triangular
4   filter, adjusting the area cMOS capture so that the risetime region lies entirely outside its
5   range.

1   22. The method of claim 1 wherein the time of detecting said step-like
2   feature is captured by:
3   connecting a digital register to the output of a digital clock; and
4   using said detecting the presence to cause said register to load the time
5   value of said clock at that instant.

1   23. The method of claim 1 wherein.
2   the filters in said filter set are analog shaping filters;
3   their output values, delayed if necessary, are captured by sample and hold
4   circuits which are triggered by the outputs of timing circuits which are started by said
5   detection event;
6   said sample and hold outputs are weighted by constants and added using
7   an analog weight-and-add circuit or multiply-and-add circuit; and
8   if additional pileup criteria are satisfied, the output of this circuit is shaped
9   by an output gating circuit so that its amplitude correctly represents the desired integrated
10   area Q_{gT}.

1   24. The method of claim 1 wherein, in addition to detecting the presence
2   of a step-like feature, an additional test is made to see if said pulse-like input signal is
3   piled up and, if it is, preventing both the following capture and determination from being
4   carried out.

1   25. The method of claim 1, further comprising capturing the peak
2   amplitudes of one or more filters and treating these captured values as members of the
3   area cMOS in determining said integrated area Q_{gT}.

1   26. The method of claim 1, further comprising, when said N-1P device is
2   not strictly a single-pole device, but has at least a second pole, plus possible additional
3   poles and/or zeros and/or a DC offset:
4   forming a mathematical model which describes the step-like output signals
5   from said N-1P device in terms of
6   i) a first (P_1) parameter that is the desired integrated area Q_{gT},

7   ii) a set of one or more second (P2) parameters that describe said N-1P

8   device's single pole response of to any previous pulse-like inputs and/or its response to

9   the finite arrival time of the present pulse, wherein the number of said P_1 and P_2

10   parameters is less than or equal to the number of sample values captured in said area

11   cMOS;

12   iii) a set of one or more third (P3) parameters that describe the non-ideal

13   portion of said N-1P device's output, including Q_{23}, its second pole response to any

14   previous pulse-like inputs, and possibly B_0, its DC offset, as well (the error terms);

15   iv) a set of one or more fixed fourth (P4) parameters that describe the

16   transfer functions of said N-1P device and of the filters in said filter set;

17   designating one or more filters in said filter set as the baseline filter set;

18   capturing, from time to time, one or more output values from said baseline

19   filter set (a baseline sample); and

20   using one or more baseline samples to update at least one type P3

21   parameter.

1   27. The method of claim 26, further comprising:

2   using said type P3 parameters to correct said integrated area Q_{gT} for the

3   effects of said error terms.

1   28. The method of claim 27 wherein said correction is accomplished by

2   adding a weighted sum of said type P3 parameters to said integrated area Q_{gT}.

1   29. The method of claim 28, when said set of P3 parameters includes only

2   Q_{23} and B_0's correcting said integrated area Q_{gT} comprises:

3   using $\overline{A}_m^{-1}[LQ_{23} + MB_0]$ as said weighted sum of said type P3 parameters,

4   wherein Q_{23} is evaluated at a time fixed with respect to said area cMOS

5   capture and $\overline{A}_m$, L, and M depend only upon type P4 parameters.

1   30. The method of claim 26 wherein at least one type P3 parameter update

2   is accomplished by:

3   using said baseline sample to produce a new estimate of said type P3

4   parameter; and

5   forming an appropriate average between said new estimate and said

6   parameter's prior value.

1   31. The method of claim 30 wherein, in forming said appropriate average:

2   the difference between said new estimate and said prior value is recorded

3   and averaged as well; and

4   this difference average is used to adjust one of the type P_4 parameters.

1   32. The method of claim 26, further comprising:
2   recording the time of capture of each baseline sample (time t_i); and
3   using t_i in said parameter updating to compensate for said P_3 parameters'
4   variation in time, so generating a time compensated parameter model (TC-model).

1   33. The method of claim 32 wherein the update of an averaged estimate
2   $<Q>$ of a time varying type P_3 TC-model parameter Q is accomplished by:
3   using said baseline sample to produce a new estimate Q_i of said parameter
4   Q at said time t_i ; and
5   averaging Q_i with said parameter's averaged estimate $<Q>_{i-1}$ at the prior
6   measurement time t_{i-1} to produce a value $<Q>_i$ of $<Q>$ at time t_i according to:

$$\langle Q \rangle_i = \frac{N-1}{N}\left(b\langle Q \rangle_{(i-1)}\right) + \frac{Q_i}{N} ,$$

8   where N is a constant and b corrects $<Q>$ for its time variation between times t_i and t_{i-1}.

1   34. The method of claim 33 wherein the type parameter $<Q>$ is associated
2   with a pole having a decay time τ_q and b is given by:

$$b = \exp\left(-\left(t_i - t_{i-1}\right)/\tau_q\right)$$

3

1   35. The method of claim 32 comprising, whenever the integrated area Q_{gT}
2   of a pulse-like input is determined, using Q_{gT} to update said TC-model parameters to
3   reflect changes in said error terms in response to said pulse-like input.

1   36. The method of claim 35 wherein:
2   a P_3 parameter Q is associated with a pole having decay time τ_q;
3   said time of determining said integrated area Q_{gT} is denoted t_i;
4   the most recent prior time of updating Q, to the value Q_{i-1}, in response to
5   capturing a baseline sample is denoted t_{i-1}; and
6   said updating of Q to value Q_i to reflect changes in said error terms in
7   response to Q_{gT} is accomplished according to:

$$Q_i = \exp\left(-\left(t_i - t_{i-1}\right)/\tau_q\right)Q_{i-1} + \beta Q_{gT} ,$$

9   where the type P_4 parameter β is the fraction of Q_{gT} which is transferred into the τ_q decay
10 mode.

1 37. In the method of claim 32, further comprising:

2 recording, in said area cMOS capture, the time of said capture;

3 adjusting said type P_3 TC-model parameters for the elapsed time Δt_i

4 between said time of area cMOS capture t_i and the last time t_{i-1} the TC-model was

5 updated; and

6 using said adjusted type P_3 parameters to correct said integrated area Q_{gT}

7 for the effects of said error terms.

1 38. The method of claim 37 wherein said integrated area correction step is

2 carried out by:

3 applying a set of weights to said type P_3 TC-model parameters; and

4 adding a sum of said weighted P_3 parameters to said integrated area Q_{gT}.

1 39. The method of claim 38 wherein.

2 said set of P3 parameters includes only Q_{23} and B_0;

3 said area cMOS comprises three captured values, designated $\sigma\Sigma+$, $\sigma\Sigma-$,

4 and σ_g, with σ_g probing the step-like feature's risetime region; and

5 said weighted sum of said area cMOS values used to determine said

6 integrated area Q_{gT}, minus said correcting weighted sum of type P_3 parameters, is given

7 by:

$$Q_{gT} = \overline{A}_m^{-1}\left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\hat{A}_m}{\overline{r}}\sigma_{g-}\right]_q - \overline{A}_m^{-1}\left[Lb_2^{\Delta t_i}\left\langle Q_{23}\right\rangle_{i-1} + M\left\langle B_0\right\rangle\right],$$

8

9 where

10 $\overline{A}_m$, K, $\hat{A}_m$, $\overline{r}$, L and M depend only upon type P_4 parameters,

11 $<B_0>$ is the TC-model estimate of B_0,

12 $<Q_{23}>_{i-1}$ is the TC-model estimate of Q_{23} at its time of last update t_{i-1}, and

13 b_2 is the amount Q_{23} exponentially decays per unit time.   -

1 40. The method of claim 26 wherein one or more of the filters whose

2 output values are captured in said area cMOS are also used in the baseline filter set.

1 41. The method of claim 26 wherein said baseline value captures are

2 allowed only at instants when said baseline filter set is not engaged in processing a step-

3 like feature in the N-1P device's output.

1 42. The method of claim 26 wherein said baseline capture is a cMOS

2 capture.

1   43. The method of claim 26 wherein:

2   said N-1P device output is digitized by an analog to digital converter;

3   the filters in said baseline filter set are digital filters;

4   the baseline sample is captured by capturing the digital outputs of said

5   filters into registers; and

6   the model parameter update step is carried out by a digital computing

7   device which reads the values captured in said registers.

1   44. The method of claim 43 wherein said digital computing device carries

2   out said correction of said integrated area Q_{gT} by adding a weighted sum of said

3   TC-model parameters to Q_{gT}.

1   45. The method of claim 26 wherein:

2   said one or more filters from which said area cMOS is captured is

3   designated the area filter set;

4   the said area and baseline filter sets are identical;

5   identical correlated multiple output samples are captured from said

6   baseline filter set (a baseline cMOS) and said area filter set (i.e., the area cMOS); and,

7   additionally,

8   said baseline cMOS captures are allowed only at instants when said

9   baseline filter set is not engaged in processing a step-like feature in the N-1P device's

10 output;

11   said area weights are also applied to the baseline cMOS values to compute

12 null area estimates of the response of said area filter set to said second pole plus possible

13 additional poles and/or zeros and/or DC offset; and

14   one or more of said null area estimates are used in said updating of at least

15 one type P3 parameter.

1   46. The method of claim 45 wherein:

2   said set of P3 parameters includes only Q_{23} and B_0;

3   said area cMOS comprises three captured values, designated $\sigma_{\Sigma+}$, $\sigma_{\Sigma-}$,

4   and σ_g, with σ_g probing the step-like feature's risetime region, and

5   said weighted sum of said area cMOS values used to determine said

6   integrated area Q_{gT} is given by:

$$Q_{gT} = \overline{A}_m^{-1}\left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\hat{A}_m}{\hat{r}}\sigma_{g-}\right]_a ;$$

7

8   values of Q_{23} are obtained from said null area measurement according to:

$$Q_{23} = \frac{1}{L}\left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\hat{A}_m}{\hat{r}}\sigma_{g-}\right]_b - \frac{M}{L}B_0$$

9

10   where $\bar{A}_m$, K, $\hat{A}_m$, $\hat{r}$, L and M depend only upon type P_4 parameters, and

11   the subscripts a and b stand for area cMOS captures and baseline cMOS captures,

12   respectively.

1   47. The method of claim 1, and further comprising, when said N-1P

2   device is not strictly a single-pole device, but has at least a second pole, plus possible

3   additional poles and/or zeros and/or a DC offset:

4   designating one or more filters in said filter set as the baseline filter set;

5   capturing, from time to time, one or more output values from said baseline

6   filter set (a baseline sample); and

7   using one or more of said baseline samples to correct said determined area

8   Q_gT for errors arising from of that portion of the N-1P device's output (the error terms)

9   which is due to said second pole plus possible additional poles and/or zeros and/or DC

10   offset.

1   48. The method of claim 47 wherein:

2   said one or more filters from which said area cMOS is captured is

3   designated the area filter set;

4   the said area and baseline filter sets are identical;

5   identical correlated multiple output samples are captured from said

6   baseline filter set (a baseline cMOS) and said area filter set (i.e., the area cMOS).

1   49. The method of claim 48 wherein:

2   said baseline cMOS captures are allowed only at instants when said area

3   filter set is not engaged in processing one of said step-like output signals from the N-1P

4   device;

5   said area weights are also applied to the baseline cMOS values to compute

6   null area estimates of the response of said area filter set to said second pole plus possible

7   additional poles and/or zeros and/or DC offset; and

8   one or more of said null area estimates are used in said correcting of said

9   determined area Q_gT.

1   50. The method of claim 49 wherein said correction of said determined

2   area Q_gT is carried out by:

3   optionally multiplying said null area estimate by a time decay factor $b^{\delta t}$,
4   where said coefficient b describes the exponential decay of said null area per unit time
5   and δt is the difference between the time of the area cMOS capture and the time of the
6   baseline cMOS capture; and

7   subtracting said null area estimate from said determined area Q_{gT}.

1   51. The method of claim 50 wherein.

2   said area cMOS comprises three captured values, designated $\sigma_{\Sigma+}$, $\sigma_{\Sigma-}$,
3   and σ_g, with σ_g probing the step-like feature's risetime region, and
4   said corrected area Q_{gT} is given by:

$$Q_{gT} = \overline{A}_m^{-1}\left[\left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\hat{A}_m}{\overline{r}}\sigma_{g-}\right]_a - b^{\delta t}\left[\sigma_{\Sigma+} - K\sigma_{\Sigma-} + \frac{\hat{A}_m}{\overline{r}}\sigma_{g-}\right]_n\right],$$

6   where the subscripts a and n label the area and null area estimates, respectively, and $\overline{A}_m$,
7   K, and $\frac{\hat{A}_m}{\overline{r}}$ are said area weights.

1   52. The method of claim 49 wherein said correction of said determined
2   area Q_{gT} is carried out by subtracting an average of several null area estimates from said
3   determined area.

1   53. The method of claim 1 wherein compensation is made for a finite time
2   extent of said pulse-like input signal by the additional steps of:
3   in response to said detecting of a step-like feature, capturing a digital
4   representation of the risetime portion of said detected step-like signal; and
5   using a digital computing device to process the values in said
6   representation in order to compute said compensation.

1   54. The method of claim 1, 3, 4, 8, 12, 13, 14, 15, 18, 19, 20, 22, 23, 32,
2   44, 45, 47 or 49 wherein said N-1P device is a charge sensitive preamplifier, said pulse-
3   like input signal is a current from a radiation detector following a radiation absorption
4   event, and said determined integrated area Q_{gT} is a charge which represents the energy
5   deposited in said detector by said absorption event.

1   55. The method of claim 1, 8, 14, 15, 20 or 47 wherein said N-1P device
2   is a superconducting bolometer, said pulse-like input signal is the flow of energy
3   following absorption of a photon in the bolometer's active volume, and said determined

4   integrated area Q_{gT} represents the energy deposited in said active volume by said photon

5   absorption event.

1   56.   Apparatus for measuring a step-like output signal from a nominally

2   single-pole device (N-1P device) in response to a pulse-like input signal to determine the

3   integrated area Q_{gT} of said pulse-like input signal, the apparatus comprising:

4   a set of one or more filters applied to the N-1P device output (the area

5   filter set);

6   circuitry that detects the presence of a step-like signal in said output signal;

7   circuitry that captures a correlated multiple output sample (the area cMOS)

8   from said area filter set in response to said detection event; and

9   circuitry that forms a weighted sum of the values in said area cMOS to

10 determine said integrated area Q_{gT} (the determined area) of said pulse-like input signal;

11   wherein the weights in said sum (the area weights) are selected to

12 compensate said area determination for errors arising either from the time structure of

13 said pulse-like input signal, or from deviations in the N-1P device's response from an

14 ideal single pole response, or from both.

1   57.   The apparatus of claim 56 wherein:

2   said N-1P device output is digitized by an analog to digital converter;

3   said filters in said area filter set are digital filters;

4   said circuitry that captures include digital registers into which the digital

5   outputs of said filters are captured; and

6   said circuitry that forms a weighted sum to determine said integrated area

7   Q_{gT} includes a digital computing device that reads the values captured in said registers,

8   multiplies each such value by an appropriate weighting constant, and adds them together

9   to compute said integrated area Q_{gT} .

1   58.   The apparatus of claim 56 wherein:

2   said N-1P device is a charge sensitive preamplifier;

3   said pulse-like input is from a radiation detector following a radiation

4   absorption event; and

5   said determined integrated area Q_{gT} represents the energy deposited in said

6   detector by said absorption event.

1   59.   The apparatus of claim 56, which further compensates for the finite

2   time extent of said pulse-like input signal, the apparatus additionally comprising:

3   circuitry that captures a digital representation of the risetime portion of

4   said detected step-like output feature in response to said detection event; and

5   a digital computing device that processes the values in said representation
6 in order to compute said compensation.

1   60. The apparatus of claim 56 wherein, in order to produce a prescribed
2 set of time relationships between the members of said area cMOS, said circuitry that
3 captures additionally includes, for each filter value to be captured, correlated sampling
4 circuitry comprising one or more of:
5   a timer that fixes the time of capture relative to said detection event; or
6   a delay element inserted between the output of said filter and said circuitry
7 that captures; or
8   wherein the output of a single filter may be captured more than once by the
9 use of multiple sets of said circuitry that captures.

1   61. The apparatus of claim 60 wherein:
2   said N-1P device is a charge sensitive preamplifier;
3   said pulse-like input is from a radiation detector following a radiation
4 absorption event; and
5   said determined integrated area Q_{gT} represents the energy deposited in said
6 detector by said absorption event.

1   62. The apparatus of claim 60 wherein:
2   said N-1P device is a superconducting bolometer;
3   said pulse-like input is the absorption of a photon in the bolometer's active
4 volume; and
5   said determined integrated area Q_{gT} represents the energy deposited in
6 said active volume by said photon absorption event.
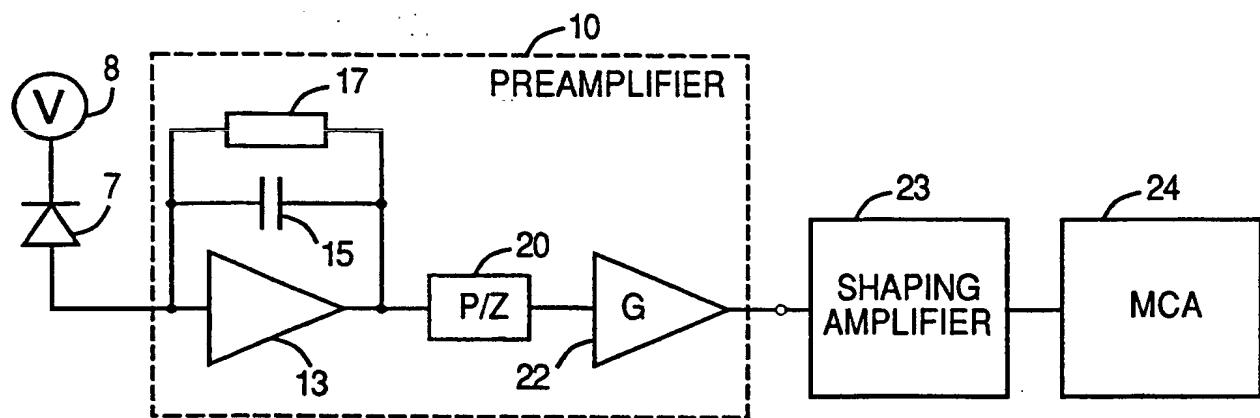
FIG. 1A
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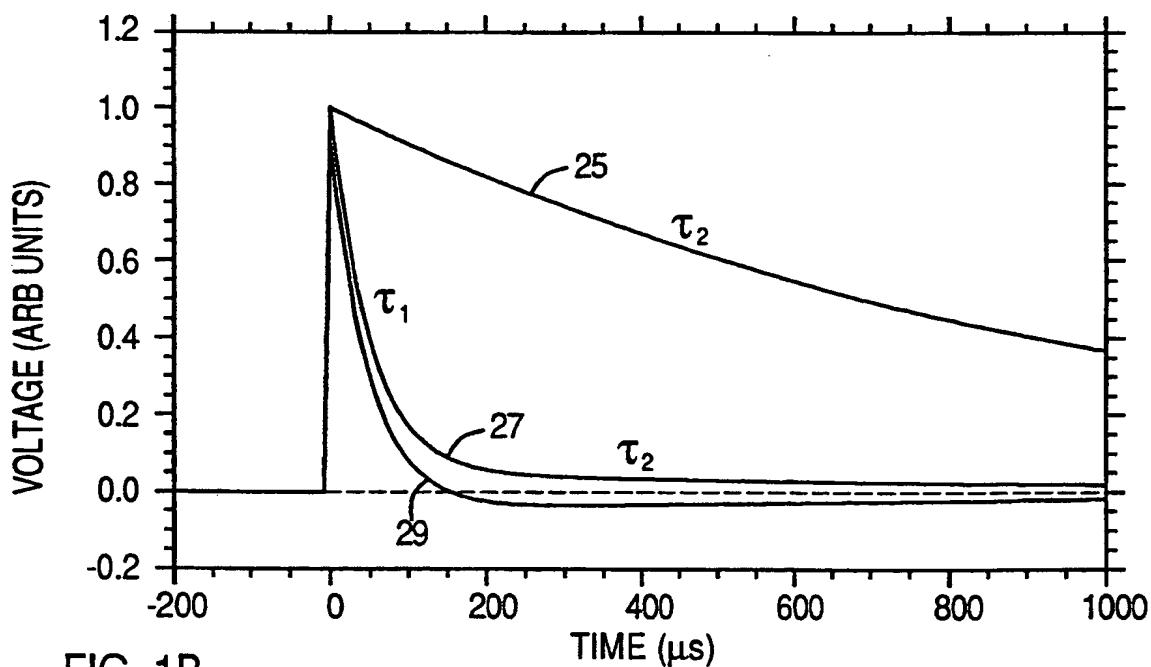
FIG. 1B
(PRIOR ART)
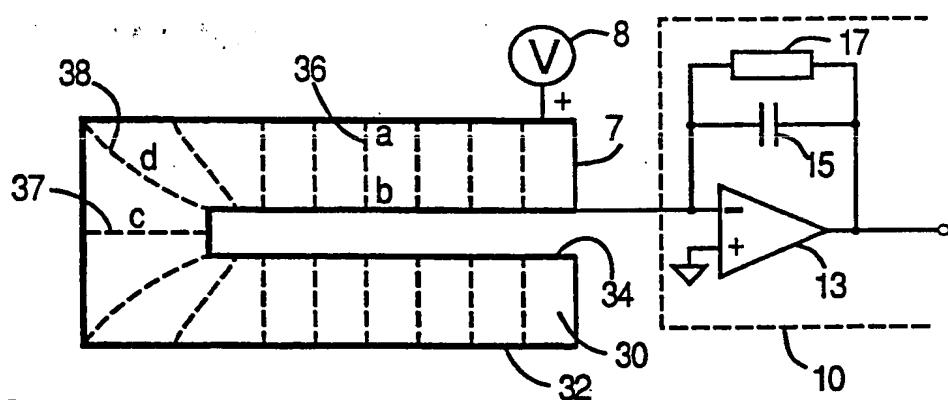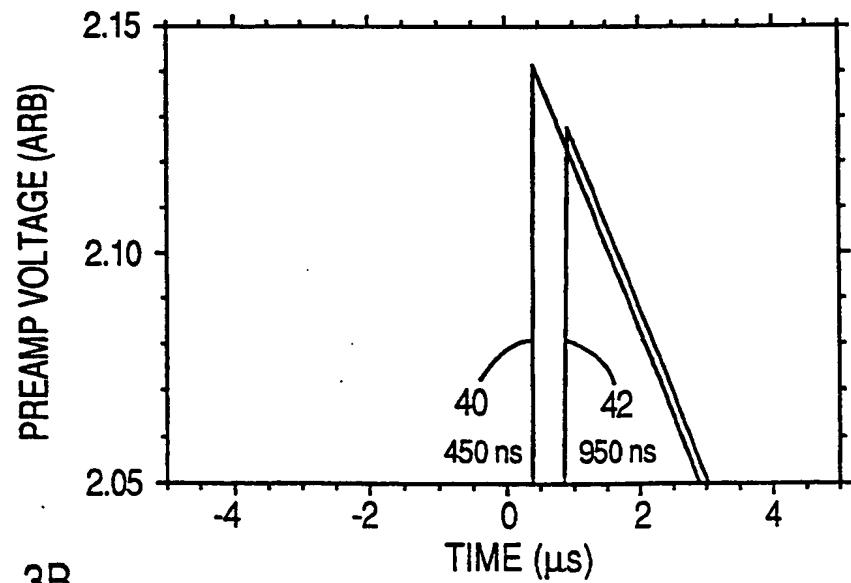
FIG. 2

FIG.3A

FIG. 3B

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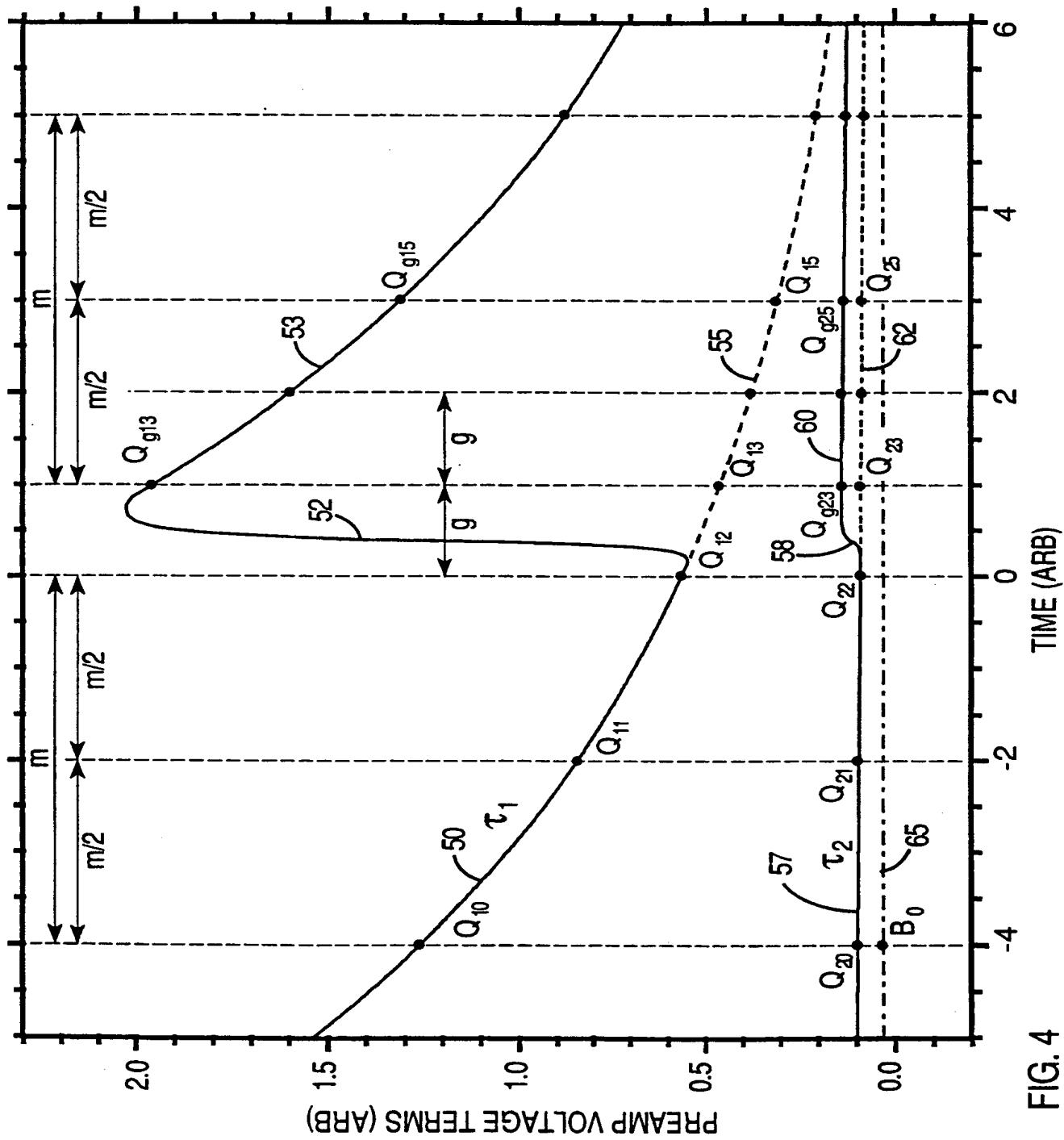
FIG. 4
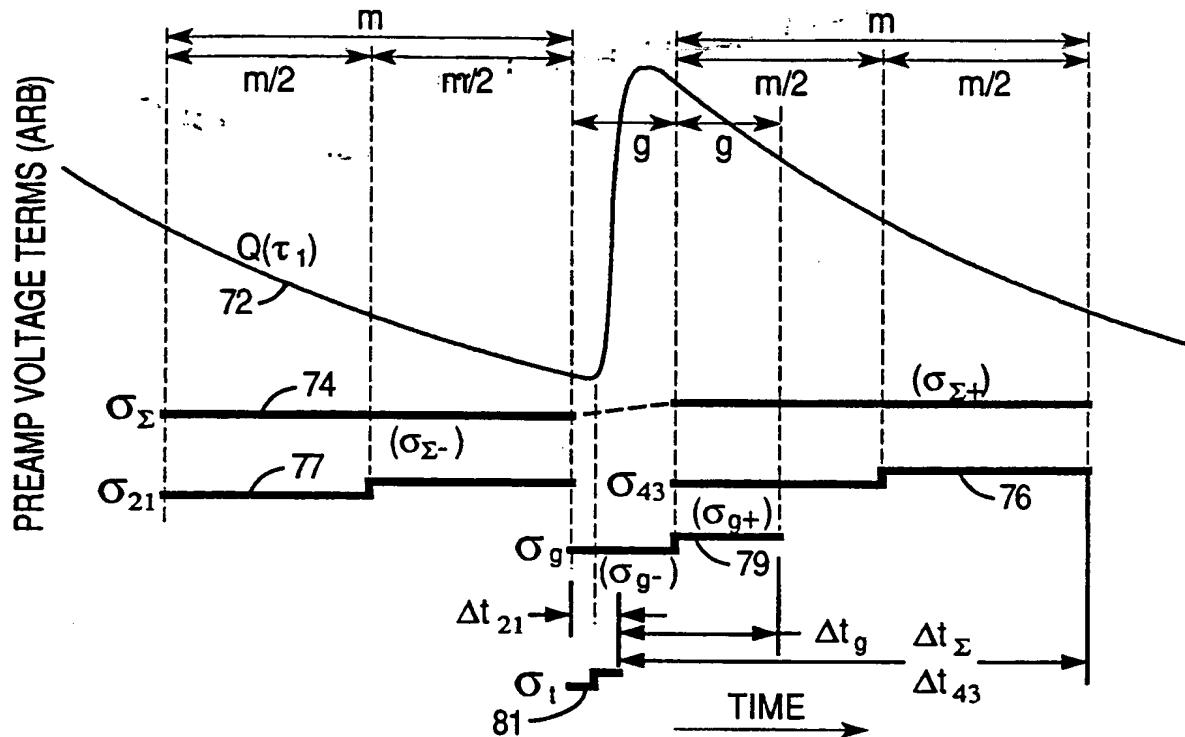
FIG. 5A
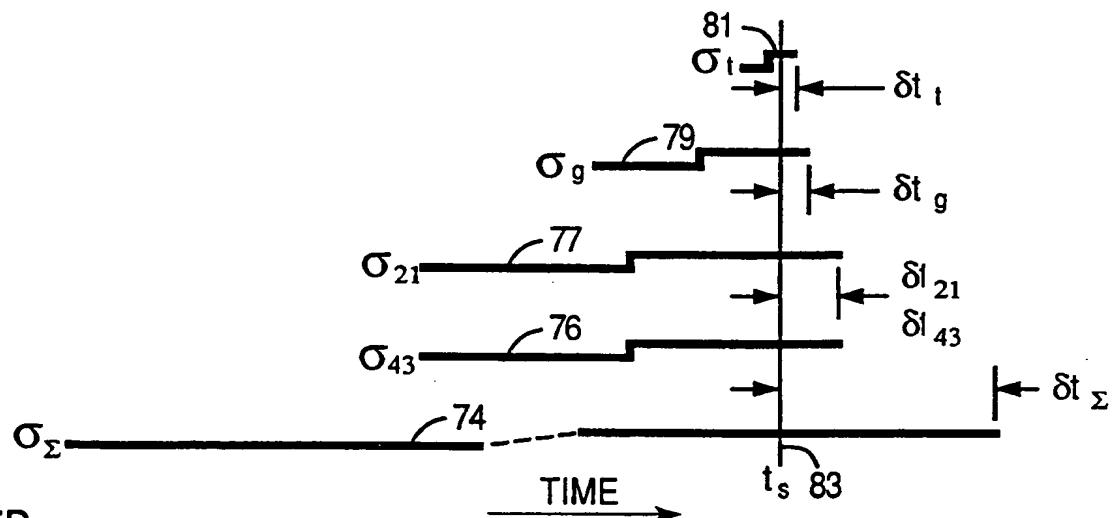
FIG.5B
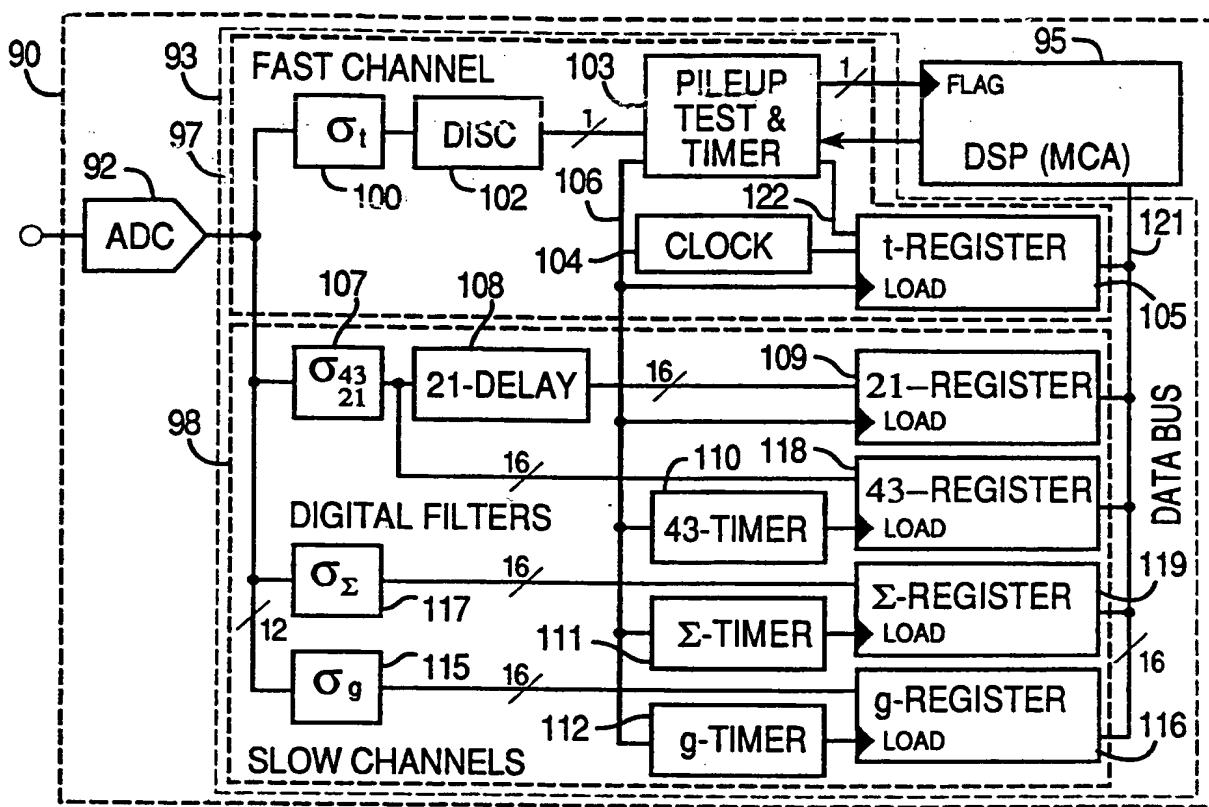
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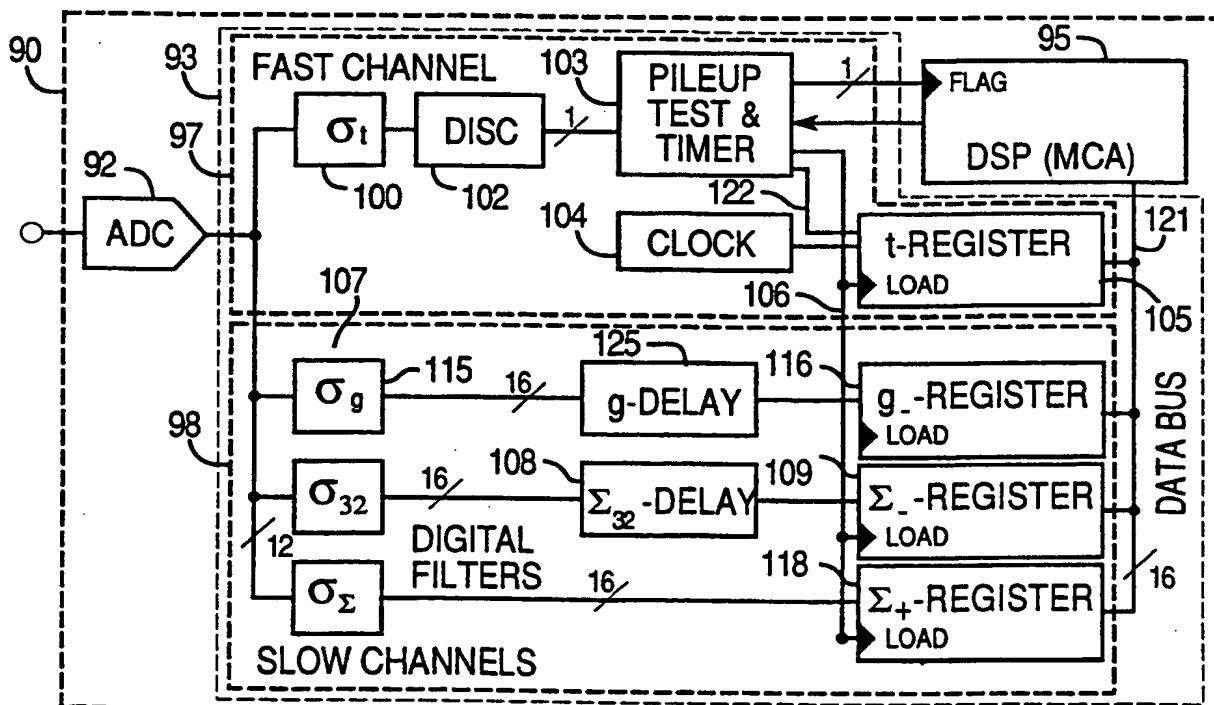
FIG.6B

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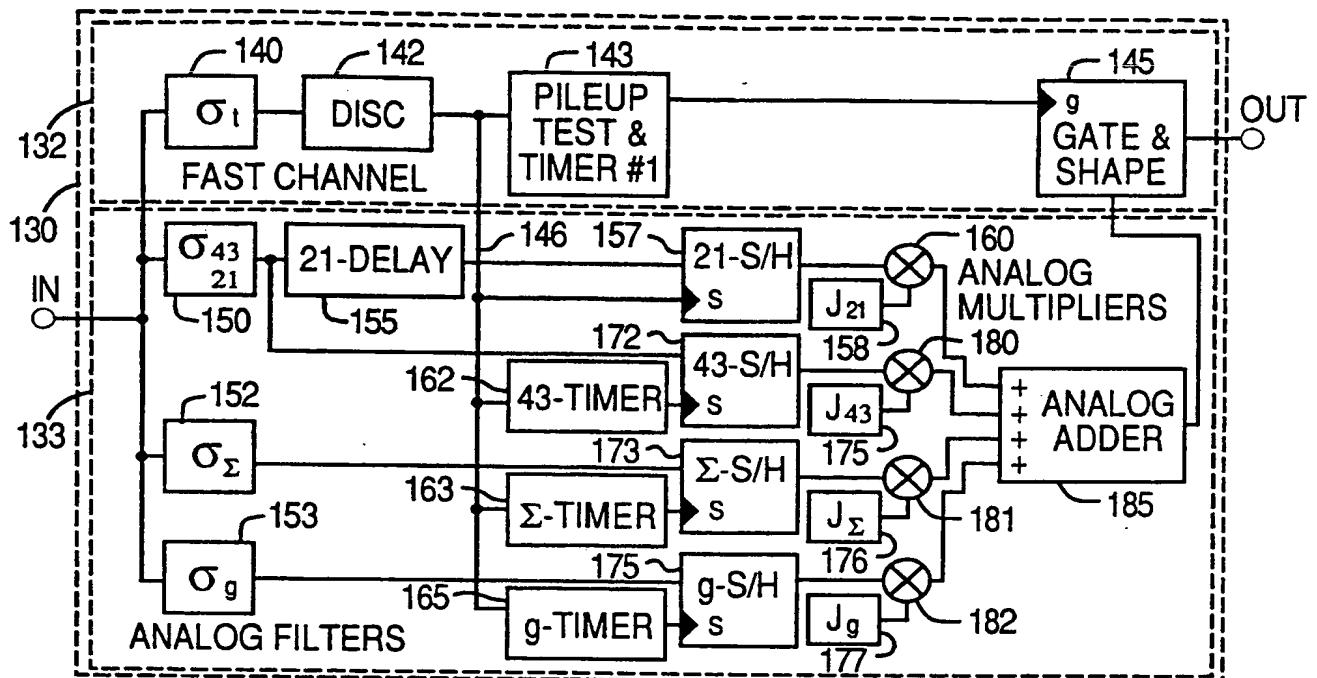
FIG. 7
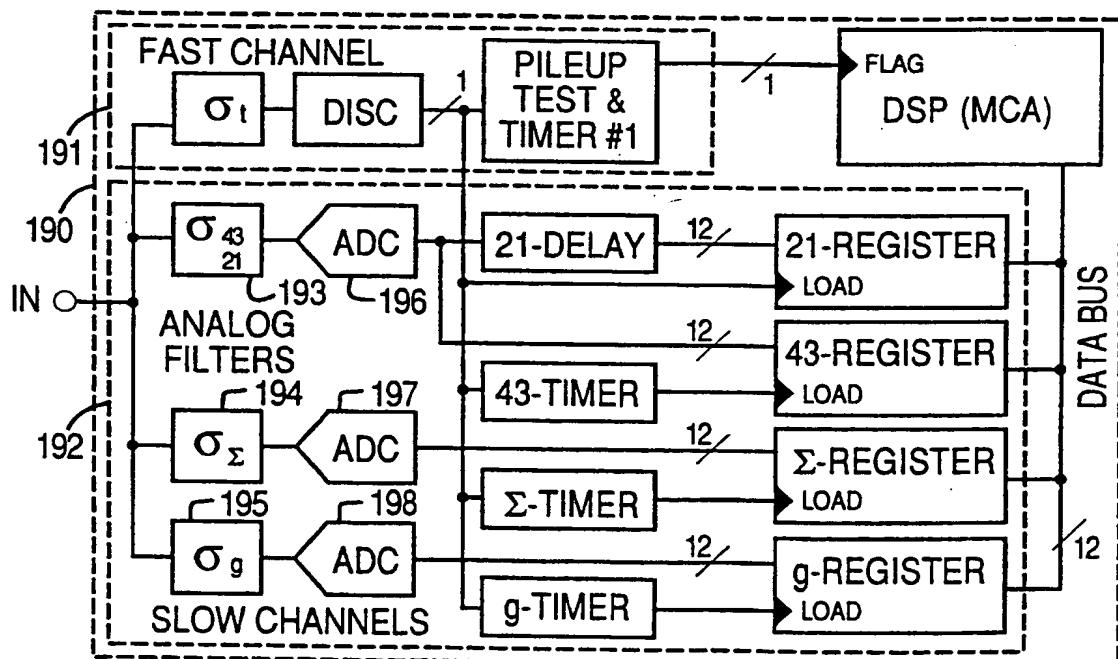
FIG. 8

INTERNATIONAL SEARCH REPORT

A.	CLASSIFICATION OF SUBJECT MATTER
IPC(7)	: HO3P 1/26
US CL	: 702/94

According to International Patent Classification (IPC) or to both national classification and IPC

B.	FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
  U.S. : 702/32,57,70,85,86,89,90,99,116,117,123,124,189-191,194,195,197; 505/161,162; 250/336.2,370.01,252.1

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)
IEEEXplore

C.	DOCUMENTS CONSIDERED TO BE RELEVANT

Category *	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
Y	US 4,314,343 A (Tomlinson) 2 February 1982 (02.02.1982), Abstract, Figure 1, column 1, line 50 - column 2, line 47.	1, 56
Y	US 4,554,633 A (Glover et al) 19 November 1985 (19.11.1985), column 9, lines 31-64.	1, 56
Y	US 5,684,850 A (Warburton et al.) 4 November 1997 (04.11.1997), column 3, lines 26-45, column 5, lines 25-35, column 9, lines 1-34, column 12, lines 42-56.	2,14,24,53,54,57-59
Y	US 5,347,129 A (Miller et al) 13 September 1994 (13.09.1994), column 2, lines 11-39, column 2, line 66 - column 3, line 12.	8
A	US 5,821,533 A (Bingham et al) 13 October 1998 (13.10.1998), column 2, lines 36-59.	1,56
A	US 4,893,018 A (Saitou) 9 January 1990 (09.01.1990), Whole document.	1-62
A	US 4,968,889 A (Hartwell et al) 6 November 1990 (06.11.1990), Whole document.	24
A	US 5,873,054 A (Warburton et al) 16 February 1999 (16.02.1999), Whole document.	1-62

☐ Further documents are listed in the continuation of Box C. ☐ See patent family annex.

*	Special categories of cited documents:	"T"	later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
"A"	document defining the general state of the art which is not considered to be of particular relevance	"X"	document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
"E"	earlier application or patent published on or after the international filing date		
"L"	document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)	"Y"	document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art
"O"	document referring to an oral disclosure, use, exhibition or other means		
"P"	document published prior to the international filing date but later than the priority date claimed	"&"	document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
	29 NOV 2000
Name and mailing address of the ISA/US	Authorized officer
Commissioner of Patents and Trademarks Box PCT Washington, D.C. 20231 Facsimile No. (703)305-3230	Marc Hoff Telephone No. 703-305-0976

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